Exercises discussed on October 25, 2011

12. Show the exponential law for formal power series,

$$\exp(ax)\exp(bx) = \exp((a+b)x), \qquad a, b \in \mathbb{K}.$$

13. Show that for the formal derivation and integration as defined in the lecture for all $a(x) \in \mathbb{K}[\![x]\!]$ it holds that

    (a) $D_x \int_x a(x) = a(x)$ (fundamental theorem of calculus I)

    (b) $\int_x D_x a(x) = a(x) - a(0)$ (fundamental theorem of calculus II)

    (c) $\langle x^n \rangle a(x) = \frac{1}{n!} \left(D_x^n a(x)\right)\big|_{x=0}$ (Taylor's formula)

14. Prove Theorem 2.9: Let $a(x) \in \mathbb{K}[\![x]\!]$. Then there exists a series $b(x) \in \mathbb{K}[\![x]\!]$ with $a(x)b(x) = 1$ if and only if $a(0) \neq 0$.

15. Define the formal power series $\sum_{n\geq 0} b_n x^n = \left(\sum_{n\geq 0} n! x^n\right)^{-1}$.

    (a) Verify that the coefficients b_8, b_9, b_{10} are negative.

    (b) Check what the *online encyclopedia of integer sequences* (OEIS) says about the sequence of coefficients $-b_1, -b_2, -b_3, \ldots$

16. Compute the first 10 coefficients in the series expansion of $a(x) = \frac{1}{1-x-x^2}$ using inversion of the formal power series $b(x) = 1 - x - x^2$. Check what the OEIS says about this sequence.