Exercises discussed on October 18, 2011

7. Show that $H_n \sim \log n$ as n tends to infinity.

(BP1) Show that the harmonic numbers cannot be expressed as a rational function, i.e., show that $H_n \notin \mathbb{C}(n)$.

8. Determine the generating function of

   (a) $a_n = n^2$.

   (b) $a_n = \frac{1}{n+1}$.

9. Show that $\left(\mathbb{K}^{\mathbb{N}}, +, \cdot\right)$ is a commutative ring with one.

10. Show that $(\mathbb{K}[\![x]\!], +, \cdot)$ is an integral domain.

11. Show that the map $D_x : \mathbb{K}[\![x]\!] \to \mathbb{K}[\![x]\!]$ defined as

$$D_x \sum_{n=0}^{\infty} a_n x^n := \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n,$$

turns $\mathbb{K}[\![x]\!]$ into a differential ring.

BP means bonus problem (i.e., not mandatory)