Exercises discussed on October 11, 2011

1. Let $f : \mathbb{Z} \to \mathbb{C}$ and $a, b \in \mathbb{Z}$ with $a \leq b$. Show that

$$\sum_{k=a}^{b} \big(f(k+1) - f(k)\big) = f(b+1) - f(a).$$

2. Determine a closed form representation for the following sums:

   (a) $\sum_{k=1}^{n} k^2$

   (b) $\sum_{k=1}^{n} k^3$

3. What is the worst-case choice of Pivot elements for Quicksort and what is the number of comparisons needed in that case?

4. Let $f : \mathbb{Z} \to \mathbb{C}$ and $a, b \in \mathbb{Z}$ with $a \leq b$ and assume that $f(k) \neq 0$ for all $a \leq k \leq b$. Show that

$$\prod_{k=a}^{b} \frac{f(k+1)}{f(k)} = \frac{f(b+1)}{f(a)}.$$

5. Determine a closed form representation for the product

$$p(n) = \prod_{k=2}^{n} \left(1 - \frac{1}{k^2}\right).$$

6. Let

$$g(n) = \sum_{k=0}^{n-1} \frac{2k}{(k+1)(k+2)}.$$

   Show that

$$g(n) = 2H_n + \frac{4}{n+1} - 4$$

   (a) by hand.

   (b) using a computer algebra system.