Exercise sheet 11

meeting on **18/06/2019**

Let $\mathcal{P}$ be a rational parametrization of an affine curve $\mathcal{C}$ defined by $f \in K[x, y]$. The *tracing index*, denoted by $\operatorname{index}(\mathcal{P})$ refers to the number of times $\mathcal{P}$ generates almost all points of $\mathcal{C}$ and can be computed for example with the formula

$$\operatorname{index}(\mathcal{P}) = \frac{\deg(\mathcal{P})}{\max(\deg_x(f), \deg_y(f))}.$$

Let $\mathcal{P}(t) = \left(\frac{\chi_{11}(t)}{\chi_{12}(t)}, \frac{\chi_{21}(t)}{\chi_{22}(t)}\right)$ be a proper parametrization. Then

$$\gcd{}_{K(\mathcal{C})[t]}(x \cdot \chi_{12}(t) - \chi_{11}(t), y \cdot \chi_{22}(t) - \chi_{21}(t)) = D_1(x, y)t - D_0(x, y)$$

for some polynomials $D_0, D_1 \in K[x, y]$ and

$$\mathcal{P}^{-1}(x, y) = \frac{D_0(x, y)}{D_1(x, y)}.$$

Exercise 42 a) Check that

$$\mathcal{P}(t) = \left(\frac{t^3 + 1}{t^2 + 3}, \frac{t^3 + t + 1}{t^2 + 1}\right)$$

is a proper rational parametrization of an affine curve $\mathcal{C}$, i.e. $\operatorname{index}(\mathcal{P}) = 1$.

b) Compute the defining polynomial of $\mathcal{C}$.

c) Compute the rational inverse $\mathcal{P}^{-1}$.