Exercise sheet 10

meeting on **04/06/2019**

Exercise 38 Let F define an irreducible curve having singularities of multiplicities $r_1, \dots, r_m$.

a) Compute the minimal degree n of F.

b) In the computation you get two possible intervals for n. Analyze the lower one and conclude whether this is a possible scenario.

c) Can you make the same conclusions if you allow reducible F?

Exercise 39 Let $\mathcal{H}(d, D)$ be the linear system of curves of degree d generated by the effective divisor D. Derive a lower bound for the dimension of $\mathcal{H}(d, D)$.

Exercise 40 Consider the linear systems of cubics $\mathcal{H}(3, D_i)$ generated by the following effective divisors.

a) $D_1 = 2 \cdot (0 : 1 : 1) + (1 : 0 : 1)$;

b) $D_2 = 2 \cdot (0 : 1 : 1) + 2 \cdot (1 : 0 : 1)$.

What is the dimension of $\mathcal{H}(3, D_i)$? Choose some curves in $\mathcal{H}(3, D_i)$ and plot them. What do you obtain?

Exercise 41 Let $\mathcal{C}$ be the irreducible projective curve defined by the form $F \in K[x, y, z]$ such that $\mathcal{C}$ is not a line and let ψ be the ring homomorphism between $K[x, y, z]$ and $\Gamma(\mathcal{C})$ such that

$$\psi(x) = \left[\frac{\partial F}{\partial x}\right], \ \psi(y) = \left[\frac{\partial F}{\partial y}\right], \ \psi(z) = \left[\frac{\partial F}{\partial z}\right].$$

a) Show that $\ker(\psi)$ is a homogeneous prime ideal. This implies that $V(\ker(\psi))$, the so-called *dual curve* of $\mathcal{C}$, is irreducible.

b) Prove that $V(\ker(\psi))$ is the algebraic closure of

$$\left\{ \left(\frac{\partial F}{\partial x}(P) : \frac{\partial F}{\partial y}(P) : \frac{\partial F}{\partial z}(P) \right) \ \middle| \ P \in \mathcal{C} \textit{ is simple} \right\}.$$

c) Compute the dual curve of $F = x^3 + y^3 - z^3$.