Exercise sheet 4

meeting on **02/04/2019**

Exercise 14 Show that if R is a Noetherian ring, then also the ring of formal power series $R[[x]]$ is Noetherian.

Exercise 15 a) Let $V(S) \subseteq \mathbb{A}^1$ be an algebraic set. Show that $V(S)$ is finite or $V(S) = \mathbb{A}^1$.

b) Find a finite basis for the ideal $\langle \{x^n + y^n \mid n \geq 1\} \rangle \subset \mathbb{C}[x, y]$.

Exercise 16 a) Let $f, g \in K[x_1, \ldots, x_n]$ and $g = g_1 g_2$ for some non-trivial factors g_1, g_2. Show that $V(f, g) = V(f, g_1) \cup V(f, g_2)$.

b) Show that $V(y - x^2, xz - y^2) = V(y - x^2, xz - x^4) =: V$. Decompose V into irreducible components and visualize it.

Exercise 17 Consider the algebraic curve $\mathcal{C}$ having the rational parametrization

$$P(t) = \left(\frac{t^5 + 1}{t^2 + 3}, \frac{t^3 + t + 1}{t^2 + 1} \right).$$

Compute the implicit defining polynomial of $\mathcal{C}$.