Exercise sheet 3

meeting on **26/03/2019**

Exercise 10 Find the Zariski closure of the following sets in $\mathbb{A}^2(\mathbb{C})$:

a) $\{(n^2, n^3) \mid n \in \mathbb{N}\}$;

b) $\{(x, y) \mid x^2 + y^2 \leq 4\}$.

Exercise 11 [Lemma 3.1.3] Let $X, Y \subseteq \mathbb{A}^n(K), S \subseteq K[x_1, \dots, x_n]$ and $a_1, \dots, a_n \in K$. Then show the following:

a) If $X \subseteq Y$, then $I(X) \supseteq I(Y)$.

b) $I(\emptyset) = K[x_1, \dots, x_n]$ and $I(\mathbb{A}^n(K)) = \{0\}$ for infinite K.

c) $I(\{(a_1, \dots, a_n)\}) = \langle x_1 - a_1, \dots, x_n - a_n \rangle$.

d) $I(V(S)) \supseteq S$ and $V(I(X)) \subseteq X$.

e) $I(X)$ is radical.

Exercise 12 Consider a planar robot with a revolute joint 1, segment 2 of length l_2, a prismatic joint 2 with settings $l_3 \in [0, m_3]$ and a revolute joint 3 with segment 4 being the hand.

a) Find appropriate joint and configuration spaces $\mathcal{J}$ and $\mathcal{C}$, respectively, and the movement mapping f in terms of trigonometric functions with the joint angles as argument.

b) Convert f into a polynomial function on a variety and then into a rational mapping.

c) Does the robot have kinematic singularities?

Exercise 13 Study the inverse kinematic problem from Example 1 in the lecture notes on robot kinematics with the following settings:

a) $l_2 = 1, l_3 = 2$;

b) $l_2 = 2, l_3 = 1$.

Interpret your results geometrically and explain the special cases.