

# Computer Analysis

Lecture Notes

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# Preface

Computer Analysis is the application of Computer Algebra to analytic problems, such as the (indefinite) integration of functions or the solution of (ordinary or partial) differential equations. The goal is to determine solution formulae for such problems.

In this lecture we describe the algebraic setting of differential rings and fields and we introduce the notions of differential polynomials, differential ideals, and (linear) differential operators.

We show how to integrate rational functions symbolically, and we describe our own recent results on explicitly solving algebraic differential equations.



# Contents

## References

1. Integration of rational functions
2. Differential rings and fields
3. The algorithm of Feng and Gao for autonomous AODEs
4. Solving non-autonomous AODEs
5. Reading further papers

## References

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