Rewriting in Computer Science and Logic
(326.065, SS 2013)
Exercises, Part 2

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Total points: 60.

Part 4. Termination

1. (4 points) Is the TRS consisting of the rewrite rules

$$\begin{aligned} d(0) &\to 0 \\ d(s(x)) &\to s(s(d(x))) \end{aligned}$$

terminating?

2. (4 points) Prove or disprove termination of the following TRSs over the signature $\mathcal{F} = \{f, a, b\}$ using the decision procedure for right-ground TRSs:

(a)

$$\begin{aligned} f(f(x,y),z) &\to f(a,f(a,b)) \\ f(a,f(x,x)) &\to f(a,f(b,a)) \\ f(a,x) &\to a \\ f(x,b) &\to f(a,a) \\ f(b,a) &\to b \end{aligned}$$

(b)

$$\begin{aligned} f(a,f(a,x)) &\to f(a,a) \\ f(x,f(a,f(x,a))) &\to f(a,f(a,f(a,f(a,b)))) \end{aligned}$$

3. (4 points) Use the polynomial interpretation $\mathcal{A}$ with $A := \mathbb{N} \setminus \{0,1,2\}$ and $P_f := X^2 + XY$ to show that the TRS

$$\{f(f(x,y),z) \to f(x,f(y,z)), \quad f(y,f(x,z)) \to f(x,x)\}$$

is terminating.

4. (4 points) Use LPO to show termination of the following TRS:

$$\{f(g(g(x)),y) \to f(g(x),f(x,y)),\quad f(g(x),g(y)) \to f(f(x,x),f(y,y))\}$$

5. (6 points) Given the TRS

$$R \coloneqq \{f(x)\odot f(y) \to f(x\odot y),\ \ f(x)\odot(f(y)\odot z) \to f(x\odot y)\odot z,\ \ (x\odot y)\odot z \to x\odot(y\odot z)\}.$$

   (a) Show termination of R using a polynomial interpretation.

   (b) Prove that termination of R can not be shown by LPO.

Part 5. Confluence

1. (5 points) Prove that the following TRSs are not convergent:

$$\begin{aligned} R_1 &\coloneqq \{f(f(x,y),z) \to f(x,f(y,z)),\quad f(x,y) \to f(y,x)\} \\ R_2 &\coloneqq \{g(0) \to 0,\quad g(s(x)) \to x,\quad g(s(s(x))) \to s(g(x))\} \\ R_3 &\coloneqq \{plus(plus(x,y),z) \to plus(x,plus(y,z)),\quad plus(x,0) \to x, \\ &\qquad plus(x,s(y)) \to s(plus(x,y))\} \end{aligned}$$

2. (5 points) Prove that the following TRS is convergent:

$$\{minus(x,0) \to x,\quad minus(0,y) \to 0,\quad minus(s(x),s(y)) \to minus(x,y)\}$$

3. (6 points) Is the TRS system

$$\{f(x,y) \to f(y,x),\quad f(f(x,y),z) \to f(x,f(y,z))\}$$

   confluent?

4. (6 points) Consider the following TRS:

$$\begin{aligned} x+0 &\to x & \qquad x-s(y) &\to p(x-y) \\ x-0 &\to x & \qquad p(s(x)) &\to x \\ x+s(y) &\to s(x+y) & \qquad s(p(x)) &\to x \end{aligned}$$

   and LPO with the precedence $+ > s$ and $- > p$. Compute all critical pairs and indicate which of them converge.

Part 6. Completion

1. (4 points) Use the basic completion procedure to complete the TRS

$$\begin{aligned} &element(cons(x,xs)) \to x \\ &element(cons(x,xs)) \to element(xs). \end{aligned}$$

   For reduction order use LPO with the precedence $member > cons$.

2. (6 points) Use the basic completion algorithm to complete the TRS

$$\begin{aligned} f(f(x)) &\to h(x) \\ f(g(x)) &\to f(x) \\ f(x) &\to g(x) \end{aligned}$$

   For reduction order use LPO with the precedence $f > g > h$.

3. (6 points) Use the improved completion procedure to complete the set of identities

$$\begin{aligned} f(f(x)) &\approx h(x) \\ f(g(x)) &\approx f(x) \\ f(x) &\approx g(x) \end{aligned}$$

   For reduction order use LPO with the precedence $f > g > h$. Show every step of the derivation, indicating the applied transformation rule.