```
<< SumCracker.m
```

SumCracker Package by Manuel Kauers — © RISC Linz — V 0.11 2009−04−05

```
ZeroSequenceQ[ChebyshevT[n + 2, x] - x ChebyshevT[n + 1, x] + (1 - x^2) ChebyshevU[n, x]]
```

True

```
(* Giving more information on the ongoing calculations *)
ZeroSequenceQ[
 ChebyshevT[n + 2, x] - x ChebyshevT[n + 1, x] + (1 - x^2) ChebyshevU[n, x], Infolevel → 2]
```

```
Creating difference ring and homomorphism...
  Extracting dependent variable...
  Normalizing exponentials...
  Cracking expression into difference system...
  Creating difference ring...
  Determining startpoint and building evaluator...
  Defining methods...
  Exiting phi constructor.
  Converting expression to difference polynomial...
Calling Zero equivalence prover...
  Entering induction loop...
  True.
```

True

```
ApproximateAnnihilator[{Fibonacci[n], Fibonacci[n + 1], (-1)^n}]
```

Q.

$$\left\{-1+(-1)^{2n},\ (-1)^n+\text{Fibonacci}[n]^2+\text{Fibonacci}[n]\,\text{Fibonacci}[1+n]-\text{Fibonacci}[1+n]^2\right\}$$

```
(* computing a linear recurrence for the given sequence *)
GetLinearRecurrence[HermiteH[2 n, x], In → n, Head → h]
```

Q(x).

$$h[2+n] == -2\left(4+12n+8n^2\right)h[n]-2\left(5+4n-2x^2\right)h[1+n]$$

```
GetLinearRecurrence[SUM[(2 k - 1) JacobiP[k, 1, 0, x], {k, 0, n}], In → n, Head → s]
```

```
Q(x).
```

$$s[3+n] == \frac{\left(42+61\,n+28\,n^2+4\,n^3\right)\,s[n]}{12+35\,n+24\,n^2+4\,n^3}+\frac{\left(-43-63\,n-28\,n^2-4\,n^3-35\,x-94\,n\,x-52\,n^2\,x-8\,n^3\,x\right)\,s[1+n]}{12+35\,n+24\,n^2+4\,n^3}+\frac{\left(13+37\,n+24\,n^2+4\,n^3+35\,x+94\,n\,x+52\,n^2\,x+8\,n^3\,x\right)\,s[2+n]}{12+35\,n+24\,n^2+4\,n^3}$$

```
(* default value for the output sequence is "SUM" (if no Head is specified) *)
```

```
GetLinearRecurrence[SUM[(2 k - 1) JacobiP[k, 1, 0, x], {k, 0, n}], In → n]
```

```
Q(x).
```

$$SUM[3+n] == \frac{\left(42+61\,n+28\,n^2+4\,n^3\right)\,SUM[n]}{12+35\,n+24\,n^2+4\,n^3}+\frac{\left(-43-63\,n-28\,n^2-4\,n^3-35\,x-94\,n\,x-52\,n^2\,x-8\,n^3\,x\right)\,SUM[1+n]}{12+35\,n+24\,n^2+4\,n^3}+\frac{\left(13+37\,n+24\,n^2+4\,n^3+35\,x+94\,n\,x+52\,n^2\,x+8\,n^3\,x\right)\,SUM[2+n]}{12+35\,n+24\,n^2+4\,n^3}$$

```
(* finding closed form representations (if possible) *)
```

```
Crack[SUM[(2 k + 1) LegendreP[k, x], {k, 0, 2 n}]]
```

```
Crack::empty : Empty SUM encountered.
```

```
Crack::empty : Empty SUM encountered.
```

```
Q(x).
```

$$\frac{1}{-1+x}\left(-\text{LegendreP}[2\,n,\,x]-2\,n\,\text{LegendreP}[2\,n,\,x]+\text{LegendreP}[1+2\,n,\,x]+2\,n\,\text{LegendreP}[1+2\,n,\,x]\right)$$

```
(* finding closed form representations in terms of some given sequences (if possible) *)
```

```
Crack[SUM[(2 k + 1) LegendreP[k, x], {k, 0, 2 n}], Into → {JacobiP[2 n, 1, 0, x], n}]
```

```
Crack::empty : Empty SUM encountered.
```

```
Crack::empty : Empty SUM encountered.
```

```
Q(x).
```

$$\text{JacobiP}[2\,n,\,1,\,0,\,x]+2\,n\,\text{JacobiP}[2\,n,\,1,\,0,\,x]$$