Prolog As A Theorem Prover

Talk in Automated Reasoning Systems

Jakob Praher

2011-06-07

Deductive Reasoning

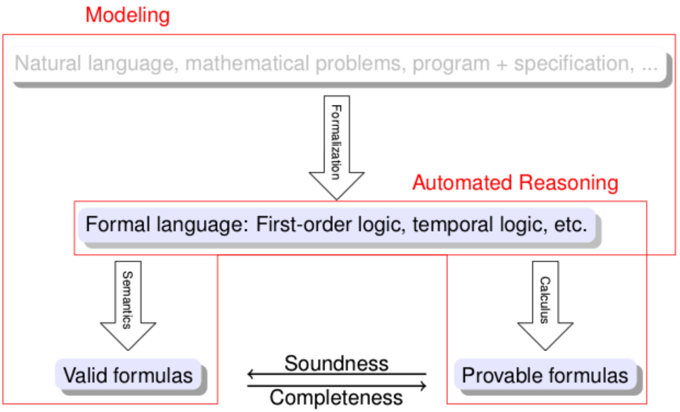


Figure: General Picture ?

Formal Deductions

- Logic Calculus
  - Set of axioms (A): Formulae assumed to be true
  - Set of formulae (Γ)
  - Rules of inference: Obtain new formuale from given ones
- Theorems of a Logic Calculus
  - The set of formulae obtained by rules of inference from $\Gamma \cup A$
  - Formal: $\{ \phi \mid \Gamma \vdash \phi \}$
  - Deduction: a set sequence of formlae recroding how ϕ was obtained from $\Gamma \cup A$
- Not unique
  - Different calculi exist (E.g. distinct sets of axioms and rules of inference)

Deductive Reasoning in Prolog

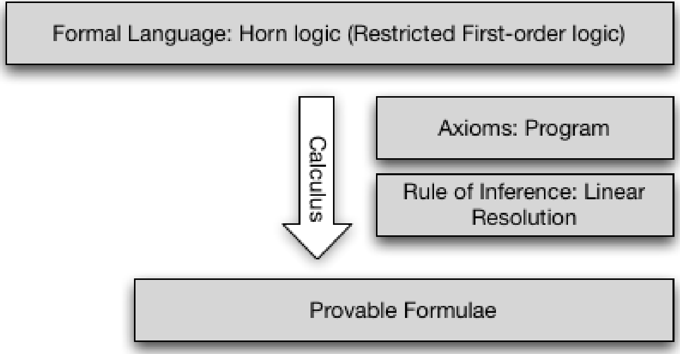


Figure: Prolog

Example Problem: Reachable Vertices in a Graph

- Situation (facts about the problem domain):

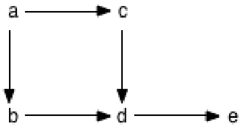


Figure: Graph with vertices (a,b,c,d,e) and directed edges.

- Problem: Is there a path starting from c?

Abstract Solution in Predicate Logic

Knowledge (Formalized situation)

edge(a,b), edge(a,c), edge(b,d), edge(c,d), edge(d,e),

$\forall_{S,E} \; edge(S,E) \Rightarrow path(S,E)),$

$\forall_{S,E} \; \exists_N \, (edge(S,N) \wedge path(N,E)) \Rightarrow path(S,E))$

Goal (Problem)

$\exists_X \, path(c, X).$

Abstract Solution as Prolog Horn Clauses

Clausal Form

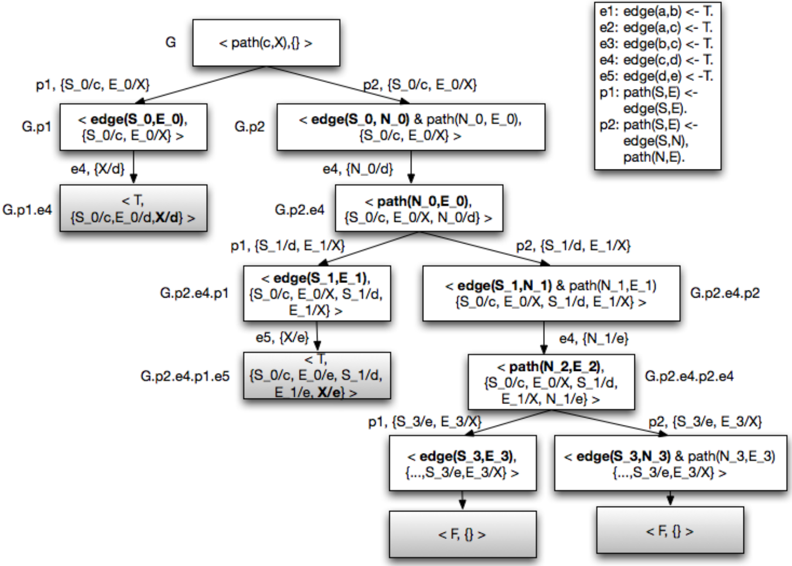
Description of situation

$$
\begin{array}{ll}
edge(a, b) \leftarrow \top & \text{(e1)} \\
edge(a, c) \leftarrow \top & \text{(e2)} \\
edge(b, d) \leftarrow \top & \text{(e3)} \\
edge(c, d) \leftarrow \top & \text{(e4)} \\
edge(d, e) \leftarrow \top & \text{(e5)} \\
path(S, E) \leftarrow edge(S, E) & \text{(p1)} \\
path(S, E) \leftarrow edge(S, N), path(N, E) & \text{(p2)}
\end{array}
$$

Problem

$path(c, X)$

Derivation Tree with Fixed Atom Selection



Limitations of Prolog as general Prover

- Formal Language: Horn Logic
  - Restricted form of first order predicate logic.
  - At most one postive literal
- Negation as failure
  - No distinction between failed derivation and something being false.
- Depth first strategy:
- Clark's completion:

Classification of Proof Methods

- Forward-reasoning (local, bottom-up)

  Start from the assumptions (axioms) until the conjecture is reached.

  - Resolution method (Robinson 1965)
  - Inverse method (Maslov, Nauk 1964)

- Goal-oriented (global, top-down)

  Start from the conjecture until we reach the axioms. Grows the tree prove tree upward.

  - Linear resolution (SLD, Prolog)
  - Model elimination method (Loveland 1968)
  - Tableau method

Full FOPL Theorem Provers in Prolog

- Prolog-like (compilation to Lisp):
  - PTTP: Prolog technology theroem prover: Uses model elimination (Loveland) (forward-reasoning)
- Lean theorem provers (Running on top of Prolog):
  - Satchmo: Tableau proof procedure (bottom-up, forward-reasoning)
  - leanTap: Lean semantic tableau theorem prover (bottom-up, forward reasoning)
  - **leanCoP**: Lean Connection-Based Theorem Prover (top-down, goal-oriented)

Connection Method Concepts

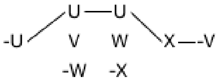
Propositional Case, Formula:

$(U \wedge V \wedge \neg W) \vee (U \wedge W \wedge \neg X) \vee \neg U \vee X \vee \neg V$

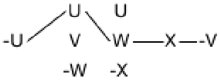
Matrix

	U	U		
-U	V	W	X	-V
	-W	-X		

Path



,



. . .

Connection Method Concepts (2)

Connection

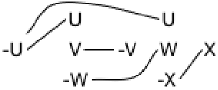
A connection in a matrix is an unordered pair of occurences of complementary literals.

Complementary Path

A connection in a matrix is an unordered pair of occurences of complementary literals.

Spanning Set of Connections

A set of connections in a matrix if every path through the matrix contains at least one of the connections belonging to this set.



,

Connection Method

Theorem

A formula of propositional logic in disjunctive normal form (DNF) is valid iff every path through its matrix representation contains connections (is complementary).

= A formuala of propositional logic in DNF is valid iff the set of all connections in its matrix is spanning.

Connection Method in First Order Logic

- Extension is done to a possible new variant of a clause (variable renaming)

  Example: $(a) \wedge (forall_x p(x) \Rightarrow p(f(x)) \Rightarrow p(f(f(a))))$
  E.g. $[p(a)], [-p(f(f(a)))], [-p(X), p(f(X))]$
- Connections must be compatible (MGU of the set of connections)
- Does not terminate on all inputs

Connection Calculus

Let (C, M, P) be (DNF-clause, set of clauses in DNF, the path).

$$\text{axiom} \frac{}{(\{\}, M, P)}$$

for some positive $C \in M$:

$$\text{start rule} \frac{(C, M \setminus C, \{\})}{M}$$

for some $L \in C, \neg L \in P$ with $\langle L, \neg L \rangle$ complementary:

$$\text{reduction rule} \frac{(C \setminus L, M, P)}{C, M, P}$$

for some $L \in C, C_1 \in M, \neg L \in C_1$ with $\langle L, \neg L \rangle$ complementary:

$$\text{extension rule} \frac{(C \setminus L, M, P) \qquad (C_1 \setminus \neg L, M \setminus C_1, P \cup \{L\})}{C, M, P}$$

Representing the Connection Calculus in Prolog

LeanCoP: Connection Calculus as Prolog Program

- Syntax: First order syntax on top of prolog structures
- Calculus: Connection calculus

References