Numerical stability of surface implicitization

Josef Schicho*, Ibolya Szilágyi**

* RICAM, Austrian Academy of Sciences, Linz, Austria
** RISC/RICAM Johannes Kepler University/Austrian Academy of Sciences Linz, Austria

josef.schicho@oeaw.ac.at,
szibolya@risc.uni-linz.ac.at

Abstract

In geometric modelling surfaces can be given essentially in two ways: implicit and parametric form. The automatic transition between the implicit and the parametric representations of surfaces is of fundamental importance. In the literature there are several symbolic/numeric implicitization techniques based on resultants [1], Gröbner–basis [2], moving surfaces [3], linear algebra [4], but the numerical stability of these algorithms is not discussed. Computing the implicit form of a surface, small changes in the input can result in large changes in the output. Using floating point numbers it is necessary to give a guarantee that the computed implicit form represents a surface close to the desired one.

In this poster we define a condition number for implicitization of surfaces. We show, that the error in the computed implicit surface, arising from the perturbation of the parametric form, is below the product of the condition number and the input error. Using the condition number we test the numerical stability of various implicitization algorithms. Here we do this for the algorithm of moving lines for cubic surfaces, and for the variation of this algorithm using singular value decomposition.

References

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