Creating an Electronic Book for Algorithms in Real Algebraic Geometry: a first experiment

Marie–Françoise Roy*
Université de Rennes I, Rennes, France

Emmanuel Briand[‡,*], Fernando Carreras[‡,*], Laureano Gonzalez–Vega[†,‡,*], Neila Gonzalez–Campos[*], Ioana Necula[†,‡,*], Herve Perdry[†,*], Natalia del Rio[‡] and Claudiu Tanasescu[†,‡,*]
Universidad de Cantabria, Santander, Spain

Fabrice Rouillier*
INRIA/LIP6 SPACES Project, Paris 6 University, Paris, France

Andreas Seidl*
Universitat Passau, Passau, Germany

Using the algorithmic nature of Real Algebraic Geometry we present first experiments on how the source book of "Algorithms in Real Algebraic Geometry" (see [1]) can be made electronic by implementing most of its algorithms and making use of algorithms from real algebraic geometry. Not only are the algorithms available for experiments (see Figure 1), but also they are intended to be integrated into the text of the book (see Figure 2) providing in this way another tool to the reader in order to get a deeper understanding of what he or she is reading and trying to understand.

Initially the following chapters and topics have been considered:

1. Chapter 8: linear algebra, Sturm, subresultant and signed subresultant sequences.
2. Chapter 9: Cauchy index and univariate Sturm queries.
3. Chapter 10: real root isolation including sign determination and Thom encoding.
4. Chapter 12: cylindrical algebraic decomposition and implicit plane curve topology.

The first seven chapters are not directly of algorithmic nature and they are devoted to several preliminaries of mathematics (algebra, real geometry, topology, etc.) and computer

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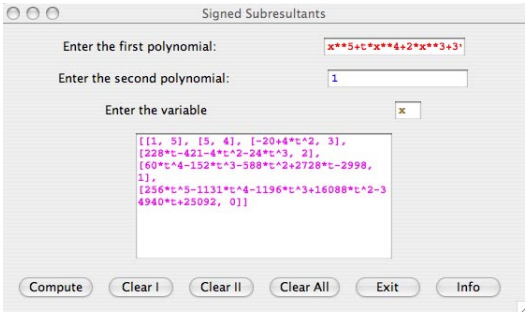

Figure 1: The `Maplet` computing Signed Subresultants.

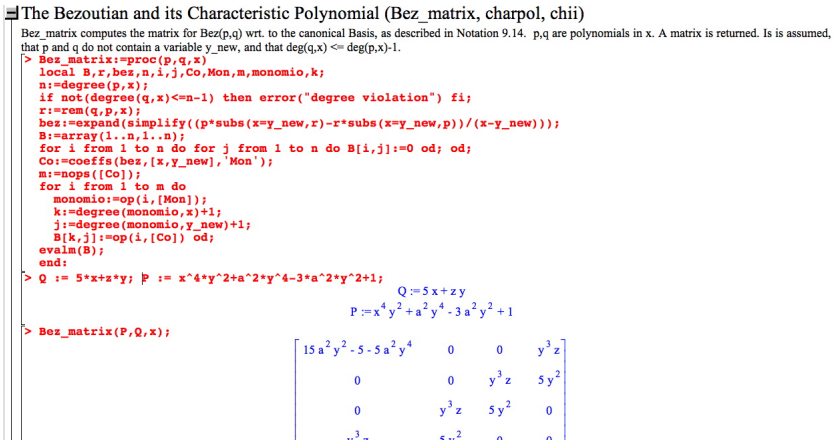

Figure 2: The `Maple` code computing the Bezoutian.

science (complexity, algorithms, etc.). The algorithms considered in this first experimental version have been implemented in the Computer Algebra System `Maple`. For most of them, the graphical interface of the algorithm has been created by using the `Maple` package `Maplets` in order to help the non `Maple` user to experiment easily (see Figure 1).

A more advanced stage of this experiment will cover Chapter 11 (Polynomial System Solving over the reals or over a real closed field) of the book [1] by Fabrice Rouillier. Subsequent versions of this electronic book will move to `TeXmacs` (the free scientific text editor, both inspired by `TeX` and `GNU Emacs` and developped by Joris van der Hoeven; see [3]) and linked to free and noncommercial computer algebra packages in order to make the electronic version of the book independent of the software used.

References

[1] Basu, S., Pollack, R., Roy, M.–F.: *Algorithms in real algebraic geometry*. Algorithms and Computation in Mathematics, 10. Springer-Verlag, Berlin, 2003.

[2] http://fgbrs.lip6.fr/~rouillie/

[3] http://www.texmacs.org/