

Creating an Electronic Book
for Algorithms in Real Algebraic Geometry:
a first experiment

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Using the algorithmic nature of Real Algebraic Geometry we present first experiments on how the source book of "Algorithms in Real Algebraic Geometry" (see [1]) can be made electronic by implementing most of its algorithms and making use of algorithms from real algebraic geometry. Not only are the algorithms available for experiments (see Figure 1), but also they are intended to be integrated into the text of the book (see Figure 2) providing in this way another tool to the reader in order to get a deeper understanding of what he or she is reading and trying to understand.

Initially the following chapters and topics have been considered:

1. Chapter 8: linear algebra, Sturm, subresultant and signed subresultant sequences.
2. Chapter 9: Cauchy index and univariate Sturm queries.
3. Chapter 10: real root isolation including sign determination and Thom encoding.
4. Chapter 12: cylindrical algebraic decomposition and implicit plane curve topology.

The first seven chapters are not directly of algorithmic nature and they are devoted to several preliminaries of mathematics (algebra, real geometry, topology, etc.) and computer

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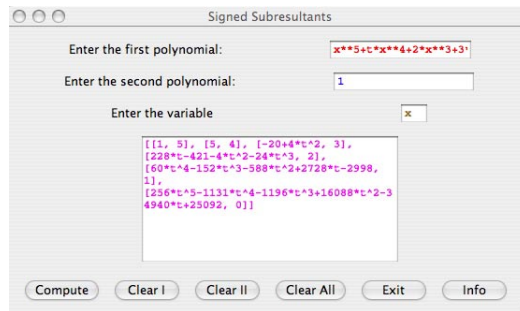


Figure 1: The Maplelet computing Signed Subresultants.

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The Bezoutian and its Characteristic Polynomial (Bez_matrix, charpol, chii)
Bez_matrix computes the matrix for Bez(p,q) wrt. to the canonical Basis, as described in Notation 9.14. p,q are polynomials in x. A matrix is returned. It is assumed,
that p and q do not contain a variable y_new, and that deg(q,x) <= deg(p,x)-1.
> Bez_matrix:=proc(p,q,x)
local B,r,bez,n,i,j,Co,Mon,m,monomio,k;
n:=degree(p,x);
if not(degrees(q,x)<=n-1) then error("degree violation") fi;
r:=rem(q,p,x);
bez:=expand(simplify((p*subs(x=y_new,r)-r*subs(x=y_new,p))/(x-y_new)));
B:=array(1..n,1..n);
for i from 1 to n do for j from 1 to n do B[i,j]:=0 od; od;
Co:=coeffs(bez,[x,y_new],Mon);
m:=nops(Co);
for i from 1 to m do
monomio:=op(i,Mon);
k:=degree(monomio,x)+1;
j:=degree(monomio,y_new)+1;
B[k,j]:=op(i,Co) od;
evalm(B);
end;
> Q := 5*x+z*y; P := x^4*y^2+a^2*y^4-3*a^2*y^2+1;
Q:=5x+zy
P:=x^4y^2+a^2y^4-3a^2y^2+1
> Bez_matrix(P,Q,x);
15a^2y^2-5-5a^2y^4  0  0  y^3z
0  0  y^3z  5y^2
0  y^3z  5y^2  0
...

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Figure 2: The Maple code computing the Bezoutian.

science (complexity, algorithms, etc.). The algorithms considered in this first experimental version have been implemented in the Computer Algebra System Maple. For most of them, the graphical interface of the algorithm has been created by using the Maple package Maplelets in order to help the non Maple user to experiment easily (see Figure 1).

A more advanced stage of this experiment will cover Chapter 11 (Polynomial System Solving over the reals or over a real closed field) of the book [1] by Fabrice Rouillier. Subsequent versions of this electronic book will move to TeXmacs (the free scientific text editor, both inspired by TeX and GNU Emacs and developed by Joris van der Hoeven; see [3]) and linked to free and noncommercial computer algebra packages in order to make the electronic version of the book independent of the software used.

References

[1] Basu, S., Pollack, R., Roy, M.-F.: *Algorithms in real algebraic geometry*. Algorithms and Computation in Mathematics, 10. Springer-Verlag, Berlin, 2003.

[2] <http://fgbrs.lip6.fr/~rouillie/>

[3] <http://www.texmacs.org/>