New methods on bimodules

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Though first developed in the ring of polynomials, the methods based on Gröbner bases also work in some noncommutative rings, e.g. the Weyl algebras or, more generally, the so-called Poincaré-Birkhoff-Witt rings (PBW, for short), including some classical quantum groups. On these generalizations, authors were mainly interested in one-sided ideals and modules, whereas methods for the two-sided counterparts are adaptations to cope with the two-sided input.

In this work we show that those methods are not necessary, due to the very well known fact that two-sided ideals and R-bimodules may be seen as left modules on the enveloping algebra $R^{\text{env}} = R \otimes R^{\text{opp}}$. First, we show that the enveloping algebra of a PBW algebra is another PBW algebra. Secondly, we find a method to move the data back and forth through the morphism

$$\mathfrak{m}^s : (R^{\text{env}})^s \longrightarrow R^s$$

in order for computations on the enveloping algebra using one-sided techniques to be carried out.

Our first contribution is a new algorithm to compute Gröbner bases for bimodules with only one call to the left Buchberger algorithm, instead of the a priori unknown number of calls typical of the aforementioned methods.

Our second contribution is the introduction and study of the *syzygy bimodule* of $F \subseteq R^s$, which is the two-sided counterpart of the left syzygy module. After finding that it has the same nice properties and applications that the later has, we are working on applications like the computation of the centralizer of a bimodule, invariant which plays a central role in noncommutative algebra.