Using Symmetry in a Polynomial System with the Dixon Resultant

Robert H. Lewis

Fordham University
http://www.bway.net/~lewis/
rlewis@fordham.edu

The Bezout-Cayley-Dixon resultant is a useful and efficient way to solve systems of polynomial equations. This has been known since at least the 1994 paper by Kapur, Saxena, and Yang [KSY]. Their key idea was proven correct in great generality by the 2000 paper of Buse, Elkadi, and Mourrain [BEM].

This paper will discuss a method to use symmetry in the set of equations to greatly speed up the computation of the (Dixon) resultant, in spite of the presence of a "spurious factor". The use of symmetry has been tried before by Gatermann [G] and Kotseiras [K], though not via resultants.

Our approach is pragmatic and experimental. A set of n equations, such as [K], may be generated for any $n > 1$. For small n the system is easily solved with Dixon. One notes that the polynomial produced by Dixon is of the form $r(x)^n s(x)$ where r is the resultant and s is the spurious factor. Using this information one may predict the spurious factor for larger n and may interpolate for the remaining polynomial r using the fact that it will be an n^{th} power.

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