Computer Algebra System SINGULAR 2-2-0

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We give a scope on algorithms and functionalities provided within SINGULAR and its extensions (non-commutative subsystem PLURAL, libraries, dynamical modules etc). Our aim is to demonstrate the highlights of the new version as well as the variety of the capabilities and applications of the system.

1 Singular's family tree: kernel, dynamic modules and librarires

We are going to show the structure of SINGULAR and its packages, extensions together with the main ideas of interplay between SINGULAR's subjects. We stress the attention on the diverse possibilities to program with SINGULAR:

- interpreter C–like language (both for interactive use and for writing a library);
- the new feature, *dynamic modules*, allowing the user to add code, written in C/C++;
- kernel features for advanced users (together with a short overview of the functions, available in the kernel).

2 New functions in Singular

We enlist a new functionalities, implemented in a new version of SINGULAR. Among others, there are

- involutive (Janet) bases computation for ideals;
- the biggest choice of ground fields on the market (with the factorization over most of them);
- Gröbner walk in kernel and library parts;

3 Plural

Constructing non-commutative algebras

We illustrate the comfortability of PLURAL's interface by constructing several important algebras with the help of three main methods we provide: generic matrices, library procedures and tensor products. For example, the procedures for setting $U(\mathfrak{sl}_n)$, $U(\mathfrak{gl}_n)$, $U(\mathfrak{g}_2)$, $U_q'(\mathfrak{so}_3)$, $U_q(\mathfrak{sl}_{\{2,3\}})$ as well as quantum affine spaces $\mathcal{O}_q(\mathbb{A}^n)$, quantum matrices $\mathcal{O}_q(M_n(\mathbb{K}))$, Weyl, Heysenberg and various finite-dimensional algebras including exterior algebras are available in the libraries.

Revised and newly–implemented algorithms

- intersection with subalgebras (elimination);
- intersection of ideals and modules;
- saturation and quotient of modules by ideals;
- algebraic dependence of pairwise commutative elements;
- center of an algebra, centralizer of a set of polynomials;
- free resolutions and Betti numbers;

Newly–developed and implemented algorithms in kernel and libraries

- correctness of a given map of G–algebras;
- preimage of an ideal/module under a morphism;
- annihilator of a Harish–Chandra module;
- central quotient and saturation of modules by ideals;
- central character decomposition of a module;

Contributions

The group of the Univerity of Granada (J. Gomez–Torrecillaz, J. Lobillo, C. Rabelo) has contributed to PLURAL libraries with the following functionalities:

- quantum matrices and quantum minors;
- Gel'fand-Kirillov dimension;