Examining a Heart on a String

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Computer algebra systems (CASes) free us from laborious calculations, we may advance much easier in research and education. Contemporary CASes can make much more than in their first years, about 25 years ago when they appeared. Nevertheless, there are still problems where CASes return erroneous answers, produce answers which are far from perfection or display correct answers in terrible form, and it may occur in relatively simple tasks. Our poster presents the case met when we investigate, e.g. in a classroom, integral curves of following ordinary differential equation of the first order

$$\ln(y) - 2x + (x/y - 2y)y' = 0,$$

where y' stands for the derivative of the function y with respect to the variable x. This exact type differential equation may be solved by most of CASes. For example, Maple 8 (from Maple Waterloo Inc.) yields the solution

$$y = \exp(-A/(2x))$$

involving Lambert W function, where

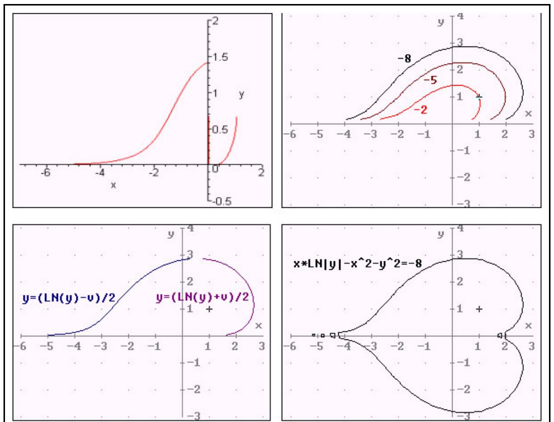
$$A = x\exp(W(-B/x)) - 2x^2 + 2c, B = 2\exp(x)^2/\exp(c/x)^2,$$

and c is a constant. Although it gives the explicit form of the function y, it is really difficult to verify its correctness (if we want to do it by hand, and not by calling the Maple function odetest). Much more friendly answer is produced by DERIVE 5, the CAS (from Texas Instruments). It returns the solution in the implicit form

$$x\ln(y) - x^2 - y^2 = c,$$

where c stands for an integral constant. It takes only a while to check the y involved in this equation satisfies considered differential equation. A problem occurs when DERIVE (as well as Maple) starts plotting the solving equation for various values of c. Plots of this equation are far from the perfection. It also takes place if we represent the solution in the form $x = x(y)$, and when we replace the factor $\ln(y)$ by $\ln(|y|)$, the graphs of transformed expressions

$$x\ln(|y|) - x^2 - y^2 = c,$$



Graphs produced in Maple 8 and DERIVE 5

Figure 1: Graphs plotted in Maple 8 (the left upper one) and in DERIVE 5 (three other ones).

$$x = (\ln(y) - \sqrt{v})/2, x = (\ln(y) + \sqrt{v})/2,$$

where $v = \ln(y)^2 - 4(y^2 + c)$, are not satisfactory, too. To reveal the real shape of curves described by these expressions we limit them as y approaches the value 0. Symbolic manipulation in DERIVE easily yields one of limits in search, but it does not handle another one, namely that concerning the right branch of the integral curve. Fortunately, a simple transformation of the expression defining this branch lets us calculate the limit at hand. In our poster we show the consecutive stages in recognizing that integral curves are heart-shaped, that our curves look like hearts on a string spread along the negative horizontal semiaxis. We point out the advances the CASes (DERIVE in particular) offer, we overcome the points they can do not treat correctly, we share our experience that lead us to final conclusion. Obviously, when doing our investigation we recall the wide world of heart-shaped curves including famous cardioid (drawn already in 16th century by Albrecht Duerer and some tens years later by Etienne Pascal), projections worked out by Rigobert Bonne in 1780, as well as shapes modelled by G.Taubin 11 years ago and by H.Dascanio in 2003.