Symbolic–Numeric Approaches for the Surface–to–Surface Intersection Problem in Computer Aided Geometric Design

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Computing the intersection curve of two surfaces is a fundamental problem in many areas, such as the CAD/CAM treatment of complicated shapes, the design of 3D objects, computer animation, NC machining and the creation of boundary representation in solid modelling (see for example [3], [4], [5], [6] or [7]). The main goal concerning the surface–to–surface intersection problem is to develop robust, accurate and fast algorithms for computing the intersection curve between two surfaces, needing the least user intervention.

We present several solutions to different instances of the surface–to–surface intersection problem. These instances are characterized either by involving real algebraic plane curves or real algebraic space curves and surfaces represented implicitly or by reducing the considered problem to a question in Computational Real Algebraic Geometry such as a Quantifier Elimination task or the determination of the topology in $\mathbb{R}^2$ of $f(x, y) = 0$ with $f \in \mathbb{R}[x, y]$ as indicated in [1] or [2].

The concrete problems whose solution is to be considered are:

- Computing the topology in $\mathbb{R}^2$ of $f(x, y) = 0$ with $f \in \mathbb{R}[x, y]$.
- Computing the intersection curve between two space surfaces represented implicitly.
- Computing the intersection curve between two space surfaces: the tangential case.
- Computing the intersection curve between two ruled surfaces in $\mathbb{R}^3$.
- Computing the intersection points between two plane curves defined implicitly: a bisection scheme.

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- Computing the intersection points between two cubic A–splines.
- Parameterizing the relative position of two ellipses.
- Eliminating geometric extraneous components when computing the implicit equation of a parametric plane curve.

The algorithms presented here have been implemented in the Computer Algebra System `Maple`. For most of them, the graphical interface of the algorithm has been created with the help of the `Maple` package `Maplets`. The accomplished implementation involves the simoultaneous use of symbolic (see [8], for example) and numerical techniques. Thus, when required, the implicitization process is done symbolically, the topology determination mentioned above involves both symbolic and numerical methods, and usually the final computations concerning the intersection curve or the intersection points are performed numerically.

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