Polynomial Algebra by Values

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This paper presents novel algorithms and novel ways of looking at some old algorithms for solving various multivariate polynomial problems. The novelty ensues from taking the point of view that polynomials are expressed by their values on a grid, not by coefficients of a monomial basis. Other authors, such as Farouk and Rajan, have looked at direct expressions of polynomial algorithms not in the monomial basis but rather in the Bernstein basis; in contrast, this poster is, we believe, the first examination of polynomial algebra by values, which is equivalent to expression in the Lagrange basis.

The point of the poster is not "improved complexity". In fact, for many algorithms, the complexity is worse than if the polynomials in question had been expressed in a monomial basis (which in the multivariate case is often quite sparse in applications). Instead, the point of the approach is to avoid possible numerical instability in unnecessary basis conversions, for example by interpolation to recover coefficients.

We present new algorithms for polynomial division (univariate and multivariate), the construction of Bezout matrices, the construction of generalized companion matrices, for GCD and some other algebraic operations. We show how to use multivariate division as the key step in a new resultant-like algorithm that encodes the (assumed zero-dimensional) variety of the problem into a generalized eigenproblem, without going through Groebner bases. We solve some example problems with this method, and show its strengths and some limitations.