Symbolic Techniques in Algebraic Multigrid Methods

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Part 1. The talk starts with a brief introduction to geometrical and algebraic multigrid methods (MGM) for solving large scale finite element equations approximating elliptic boundary value problems. In contrast to the geometrical MGM that is based on a hierarchy of finer and finer meshes, the algebraic multigrid (AMG) method needs only single (fine) grid information, usually the matrix and the right-hand side of the system that is to be solved. In the AMG, the hierarchy of coarser and coarser representation of the fine grid problem must be generated algebraically. There are very efficient coarsening techniques for systems where system matrix is an M-matrix.
In the paractically important case where the original system matrix is not an M-matrix, we construct an auxiliary fine grid matrix that is an M-matrix and contains all characteristical features of the original matrix. Then the coarsening is controlled by this auxiliary M-matrix. One technique of constructing this auxiliary M-matrix makes use of the element stiffness matrices of the original finite element stiffness matrix. Solving an constraint optimization problem for each finite element stiffness matrix, we find that element M-matrix that is as close as possible to the finite element stiffness matrix in the spectral sense.
Part 2. In general, we have to solve as many small constraint optimization problems as we have finite elements in our discretization. The numerical solution of all these optimization problems can be very expensive. In order to speed up the solution of these sub-problems, we solve the optimizations symbolically once and for all, and then instantiate the solutions by the local data coming from the finite elements. This accelarates the AMG solution process considerably.
In the 2-dimensional case (linear triangular elements), the symbolic solution can be computed without difficulties. The 3-dimensional case (linear tetrahedral elements) is harder because the number of variables is too large

in order to apply the available methods (Gröbner bases, resultants). However, one can get rid of several variables by re-formulating the problem in a different way, and this re-formulated problem has a short symbolic solution.