Numerical computation of Gröbner bases

Aleksey Kondratyev
Research Institute for Symbolic Computation,
Johannes Kepler Univesität, Linz, Austria.
E-mail: Aleksey.Kondratyev@risc.uni-linz.ac.at

We present an installation of Buchberger's algorithm for the numerical computation of reduced Gröbner bases of a multivariate polynomial system. We are mainly interested in the case of zero-dimensional systems although the implemented algorithm can be applied in a general situation. If the system is zero-dimensional then the result of the Gröbner basis computation is used to form a multiplication matrix of the quotient ring and to solve the system numerically according to [2]. Our work is based on the installation F_4 of Buchberger's algorithm developed by J. C. Faugere [1]. We have kept the numerically attractive features of F_4 (extensive use of linear algebra and the collection of all operations with coefficients into row echelon reductions of the elimination matrix [1]); but we have improved the numerical stability of Gröbner basis computation by developing extensive selection strategies for critical pairs and reduction, and a technique for the delayed treatment of certain relations. We use a pool of superfluous relations appearing in the flow of the algorithm which are not inserted to the Gröbner basis but used by the reduction strategy routine to prevent the growth of the total degrees and of the spread in the magnitude of the coefficients in the intermediate polynomials. All selection strategies are heuristic and based on $1-$step-in-depth analysis of available choices.

Our implementation is written in C++ using idioms for emulation of symbolic language styles. It supports hardware floating point (single and double precision) as well as extra-long software emulated floating point as a domain for coefficients.

References

[1] *J. C. Faugere, A new effcient algorithm for computing Grobner basis F_4.* Technical report, LIP6, 1998.

[2] *H. J. Stetter*, *Eigenproblems are at the heart of polynomial system solving.* SIGSAM Bulletin, 30(4), 22–25, 1996.