Differential Equations from an Algebraic Standpoint

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Differential algebra was introduced by J.F. Ritt in the 30s as an extension of commutative algebra to differential equations. The basis theorem and the Nullstellensatz find their generalisation in this context.

Recent development have lead to effective algorithms to compute good representations of radical differential ideals generated by a finite family of differential polynomials. This representation is given as an intersection of differential ideals well defined by their characteristic sets. This representation allows to test membership to the radical differential ideal.

In other words, for a given system of ordinary or partial differential equations, the algorithms will first decide if there exist solutions. If there are, the algorithm will output a finite set of differential systems with a triangular form such that the set of solutions of the original system is equal to the union of the non singular solution set of the output systems.

With this representation, we can answer typical differential elimination questions, for instance:

- do the solutions of a system satisfy an ordinary differential equation?
- what are the differential equation satisfied by a subset of the unknown functions?
- what are the algebraic constraints?

We shall illustrate how these question arise in diverse topics.

I will also show how differential algebra contributes to the study of singular solutions of a single differential equation. For instance, for first order differential equations, we can read on the algebraic structure of the equation whether a singular solution is an envelope or a limit case of the nonsingular solutions.