Numerical Treatment of Overdetermined Linear PDEs

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In the first part of this talk, we show how index concepts and the notion of an underlying equation for systems of differential algebraic equations (DAEs) can be generalised to systems of overdetermined linear first order partial differential equations using the language of formal theory for PDEs. The central idea is to complete to involution the systems under consideration; if this is carried out with the perturbed system, one obtains an estimation of the perturbation index in terms of determinacy and involution indices which can be seen as refinements of differentiation indices. For involutive systems of PDEs in so-called Cartan normal form, the equations of highest class constitute the underlying equation, whereas the equations of lower class can be considered the constraints.

In the second part, we apply these methods to a certain class of linear systems called weakly overdetermined systems and examine their behaviour under semi-discretisations. Like for DAEs, the drift off the constaints can be measured for such systems. For both finite differences and spectral methods we can show that the resulting DAE is formally integrable if and only if the original system has been in involution. This inhibits the existence of hidden constraints. Finally, for general sytems we relate the index of the discretised system to certain values appearing in the completion process. This result holds also for general linear systems.

References

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