Spreadsheet View and Basic Statistics Concepts

GeoGebra 3.2 Workshop Handout 9

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Table of Contents

1. Introduction to GeoGebra's Spreadsheet View

The dynamic mathematics software GeoGebra provides three different views of mathematical objects: a *Graphics View*, a, numeric *Algebra View* and a *Spreadsheet View*. They allow you to display mathematical objects in three different representations: graphically (e.g., points, function graphs), algebraically (e.g., coordinates of points, equations), and in spreadsheet cells. Thereby, all representations of the same object are linked dynamically and adapt automatically to changes made to any of the representations, no matter how they were initially created.
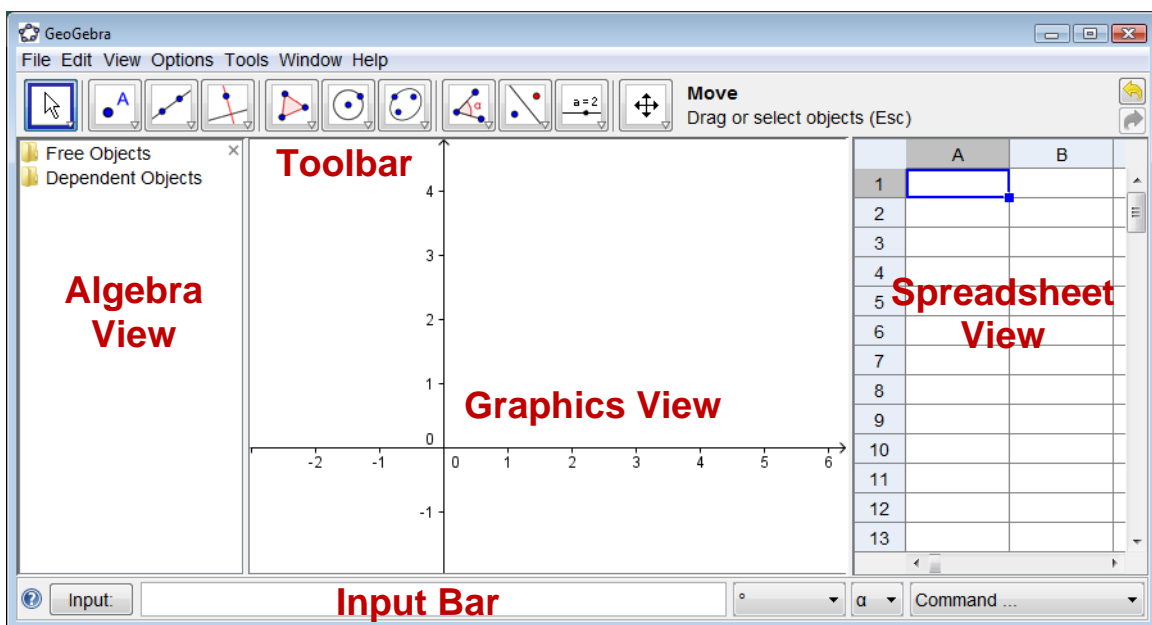
Geometric Constructions

Using the **construction tools** available in the *Toolbar* you can do geometric constructions in the *Graphics View* with the mouse. Select any construction tool from the *Toolbar* and read the *Toolbar Help* (next to the *Toolbar*) in order to find out how to use the selected tool. Any object you create in the *Graphics View* also has an algebraic representation in the *Algebra View*.

Note that you are able to **move objects in the *Graphics View*** by dragging them with the mouse. At the same time, their algebraic representations are dynamically updated in the *Algebra View*.

Every icon in the *Toolbar* represents a *Toolbox* that contains a selection of similar construction tools. In order to open a *Toolbox*, you need to click on the small arrow in the lower right corner of the *Toolbar* icon.

Hint: Construction tools are organized by the nature of resulting objects or their functionality. You will find tools that create different types of points in the *Point toolbox* (default icon •^A) and tools that allow you to apply geometric transformations in the *Transformation toolbox* (default icon •⟍).

Algebraic Input and Commands

Using the *Input Bar* you can **directly enter algebraic expressions** in GeoGebra. After hitting the *Enter*-key your algebraic input appears in the *Algebra View* while its graphical representation is automatically displayed in the *Graphics View*. For example, the input `f(x) = x^2` gives you the function *f* in the *Algebra View* and its function graph in the *Graphics View*.

In the *Algebra View*, mathematical objects are organized as **free** or **dependent objects**. If you create a new object without using any other existing objects, it is classified as a *free object*. If your newly created object was created by using other existing objects, it is classified as a *dependent object*.
Hint: If you want to hide the algebraic representation of an object in the *Algebra View*, you can specify the object as an **Auxiliary Object**: Right click (MacOS: *Ctrl*-click) on the corresponding object in the *Algebra View* and select 'Auxiliary Object' from the appearing context menu. By default, Auxiliary Objects are not shown in the *Algebra View*, but you can change this setting by selecting the item 'Auxiliary Objects' from the *View* menu.

Note that you are able to **modify objects in the *Algebra View*** as well: Make sure that you activate the ⬉ *Move* tool before you double click on a *free object* in the *Algebra View*. In the appearing textbox you can directly edit the algebraic representation of the object. After hitting the *Enter*-key, the graphical representation of the object will automatically adapt to your changes.
If you double click on a dependent object in the *Algebra View*, a dialog window appears allowing you to redefine the object.

GeoGebra also offers a wide range of **commands** that can be entered into the *Input Bar*. You can open the list of commands in the right corner of the *Input Bar* by clicking on the button *Command*. After selecting a command from this list (or typing its name directly into the *Input Bar*) you can press the *F1*-key to get information about the syntax and arguments required to apply the corresponding command.

Spreadsheet Cells Input

In GeoGebra's *Spreadsheet View* every cell has a **specific name** that allows you to directly address each cell. For example, the cell in column A and row 1 is named *A1*.

<u>Note</u>: These cell names can be used in expressions and commands in order to address the content of the corresponding cell.

In the spreadsheet cells you can **enter** not only numbers, but **all types of mathematical objects** that are supported by GeoGebra (e.g., coordinates of points, functions, commands). If possible, GeoGebra immediately displays the graphical representation of the object you enter into a spreadsheet cell in the *Graphics View* as well. Thereby, the name of the object matches the name of the spreadsheet cell used to initially create it (e.g., *A5, C1*).

<u>Note</u>: By default, spreadsheet objects are classified as *Auxiliary Objects* in the *Algebra View*. You can show or hide these *Auxiliary Objects* by selecting 'Auxiliary Objects' from the *View* menu.

Customize the User Interface and Toolbar

The **user interface** of GeoGebra can be customized by using the *View* menu. For example, you can hide different parts of the interface (e.g., the *Algebra View*) by unchecking the corresponding menu item in the *View* menu (e.g., 'Algebra View').
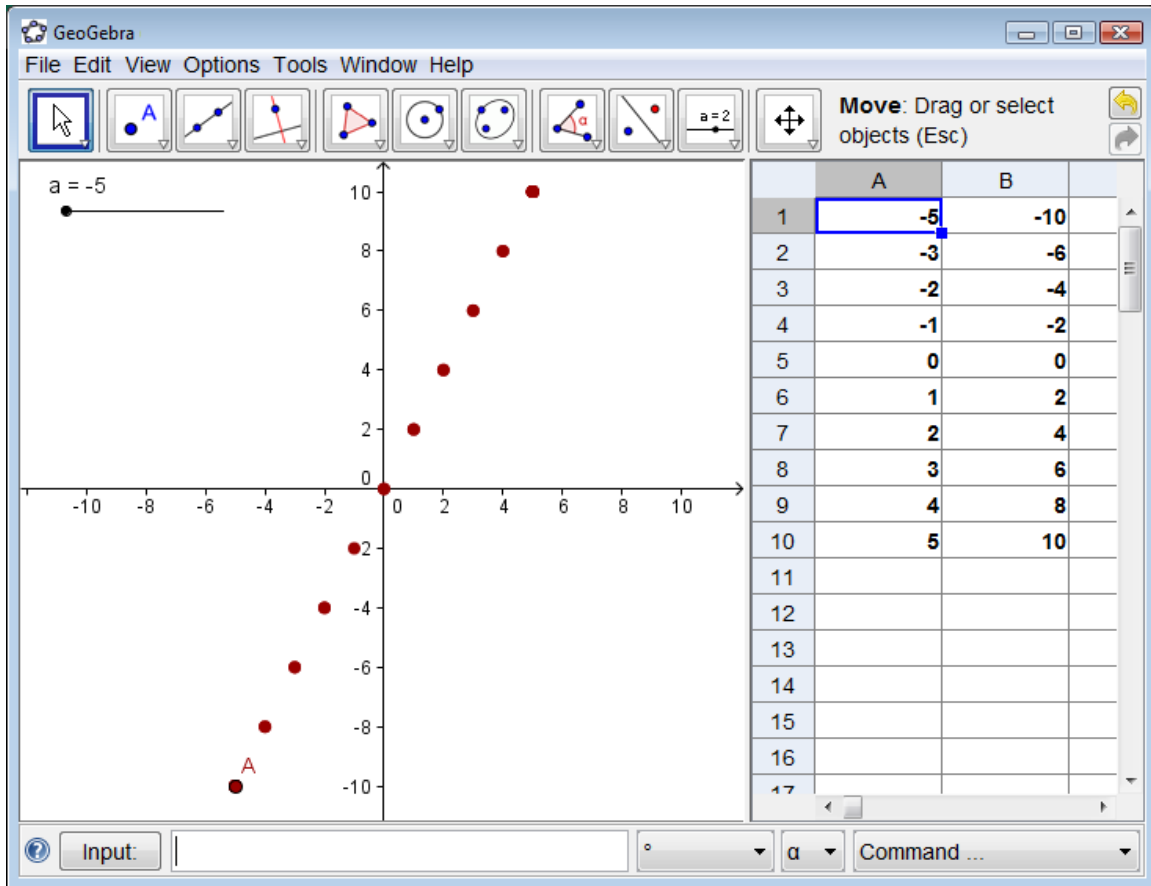<u>Note</u>: In this way, you can also show or hide the coordinate axes and a coordinate grid.

The **Toolbar** can be customized by selecting 'Customize Toolbar…' from the *Tools* menu. Highlight the tool or toolbox you want to remove from the *Toolbar* in the list on the left hand side of the appearing dialog window and click button 'Remove >' in order to remove the tool/toolbox from the *Toolbar*. You can restore the default *Toolbar* by clicking on the button 'Restore Default Toolbar' in the left lower corner of the dialog window.

# 2. Record to Spreadsheet Feature

Preparations

- Open a new GeoGebra file.
- Show the *Spreadsheet View* (*View* menu).
- Hide the *Algebra View* (*View* menu).

Instructions

1	a = 2	Create a slider with default interval and increment 1. Hint: Select tool *Slider* and click in the *Graphics View* to set the position for the slider. In the appearing dialog window change the increment to 1 and click the *Apply* button.
2	A = (a, 2a)	Create point *A* by entering `A = (a, 2a)` into the *Input Bar*. Hint: The value of slider *a* determines the *x*-coordinate of point *A* while the *y*-coordinate is a multiple of this value.
3	A A	Show the label of point *A* in the *Graphics View*.
4		Change the value of slider *a* to examine different positions of point *A*.
5		Use tools *Move Drawing Pad*, as well as *Zoom In* and *Zoom Out* to adjust the visible part of the *Graphics View* and make point *A* visible in all positions.

6		Turn on the trace of point *A*. Hint: Right click (MacOS: *Ctrl*-click) on point *A* and select 'Trace On' from the appearing *Context Menu*.
7		Change the value of slider *a* to examine the trace point *A* leaves for every slider position.
8		Set the value of slider *a* to -5.
9		Record the coordinates for different positions of point *A* to the spreadsheet: (1) Select tool *Record to Spreadsheet*. Then, click on point *A* in order to highlight it. Note: The coordinates for the actual position of point *A* are immediately entered into cells *A1* (*x*-coordinate) and *B1* (*y*-coordinate) of the spreadsheet. (2) Now, change the value of slider *a* in order to record the coordinates of all other possible positions of point *A* to the spreadsheet as well. Note: Do not switch to another tool before moving the slider.

Additional Tasks

Task 1: Examine the pattern of y-values in column *B*

You could give this construction to your students and let them explore the pattern in column *B*, which is created by the *y*-coordinates of different positions of point *A*. Encourage your students to make a prediction about a function graph that runs through all different positions of point *A*. Have your students enter the corresponding function into the *Input Bar* in order to check whether their prediction was correct (e.g., students enter `f(x) = 2x` to create a line through all points).

Task 2: Create a new problem

Change the *y*-coordinate of point *A* in order to create a new problem:
- Right click (MacOS: *Ctrl*-click) on point *A* and select 'Properties…' from the appearing *Context Menu*.
- In tab 'Basic' you can change the *y*-coordinate of point *A* in the text field 'Definition' to, for example, `a^2`.
- Use the other tabs of the *Properties Dialog* in order to change the color (tab 'Color') or size (tab 'Style') of point *A*.
- Close the *Properties Dialog* when you have made all desired changes.

- Repeat steps 7 to 9 of the instructions above in order to record the coordinates of the new positions of point *A* to the spreadsheet.
  Note: If you did not delete the old values in columns *A* and *B*, GeoGebra automatically uses the next two empty columns (e.g., columns *C* and *D*) in order to record the new values for *x*-coordinates and *y*-coordinates.

3. Relative Copy and Linear Equations

Preparations

- Open a new GeoGebra file.
- Show the *Spreadsheet View* (*View* menu).
- Hide the *Algebra View* (*View* menu).

Instructions

1	✥	Activate tool *Move Drawing Pad* and drag the origin of the coordinate system close to the lower left corner of the *Graphics View*.
2	(0, 0)	In the *Spreadsheet View*, click on cell *A1* and enter the point coordinates (0, 0).
3	(1, 1)	In the *Spreadsheet View*, click on cell *A2* and enter the point coordinates (1, 1).
4	A A	Select tool *Show/Hide Label* and show the labels of both points in the *Graphics View*.
5	▷	Relative copy the inserted point coordinates to other cells in column *A*: (1) Highlight both cells *A1* and *A2* by using the mouse. (2) Click on the little square at the lower right corner of the highlighted cell range. (3) Hold the mouse button down and drag the pointer down to cell *A11*.
6	✥ 🔍 🔍	Use tools *Move Drawing Pad*, as well as *Zoom In* and *Zoom Out* to adjust the visible part of the *Graphics View* and make all points visible.

Additional Tasks

Task 1: Examine the coordinates of the point sequence

What sequence of numbers is created if you apply the 'relative copy' feature of the GeoGebra spreadsheet the way it is described above?
Hint: Examine the *x*-coordinates of all created points and come up with a conjecture about how they are related. Then, check your conjecture using the *y*-coordinates of the points.

Task 2: Find the matching equation

Make a prediction about an equation that would create a graph going through all points of this sequence. Enter this equation into the *Input Bar* in order to check your prediction.

Task 3: Create a new problem

Change the coordinates of the initial points in order to create a sequence of points that can be examined by your students.

<u>Version 1</u>: Change the initial points in the *Spreadsheet View*.
Double click in cell *A2* and change the coordinates of the corresponding point to `(1, 2)`. After hitting the *Enter*-key, all points that depend on point *A2* automatically adapt to this change, both in the *Spreadsheet View* as well as in the *Graphics View*.

<u>Version 2</u>: Change the initial points in the *Graphics View*.
Activate tool   *Move* and drag point *A2* to a different position in the coordinate system. Immediately, all dependent points dynamically adapt to these changes both in the *Graphics View* as well as in the *Spreadsheet View*.
<u>Note</u>: In order to restrict the coordinates of the points to be integers, you can change the option 'Point Capturing' in the *Options* menu to 'On (Grid)'. You may also display the coordinate grid by selecting 'Grid' from the *View* menu.

<u>Hint</u>: By changing the coordinates of point *A1* as well, you are able to create problems that result in linear equations of the form $y = m\,x + b$ which do not run through the origin of the coordinate system.
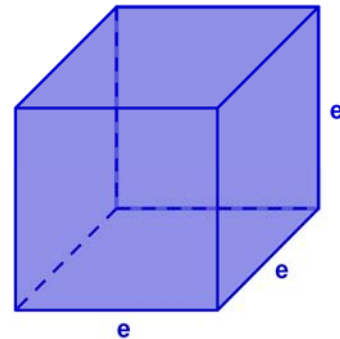
4. Investigating Number Patterns

Let's investigate how the surface of a cube changes depending on the length of its edges.

Preparations with Paper and Pencil

Calculate the surface of a cube for the given length *e* of its edges. Pick at least two edge lengths from each table but do not pick the same numbers as your neighbor.

Edge	Surface	Edge	Surface
1		6	
2		7	
3		8	
4		9	
5		10	

Preparations in GeoGebra

- Open a new GeoGebra file.
- Show the Spreadsheet View (*View* menu).
- Hide the *Algebra View* (*View* menu).
- In the *Options* menu set the *Labeling* to 'New Points Only'*.*
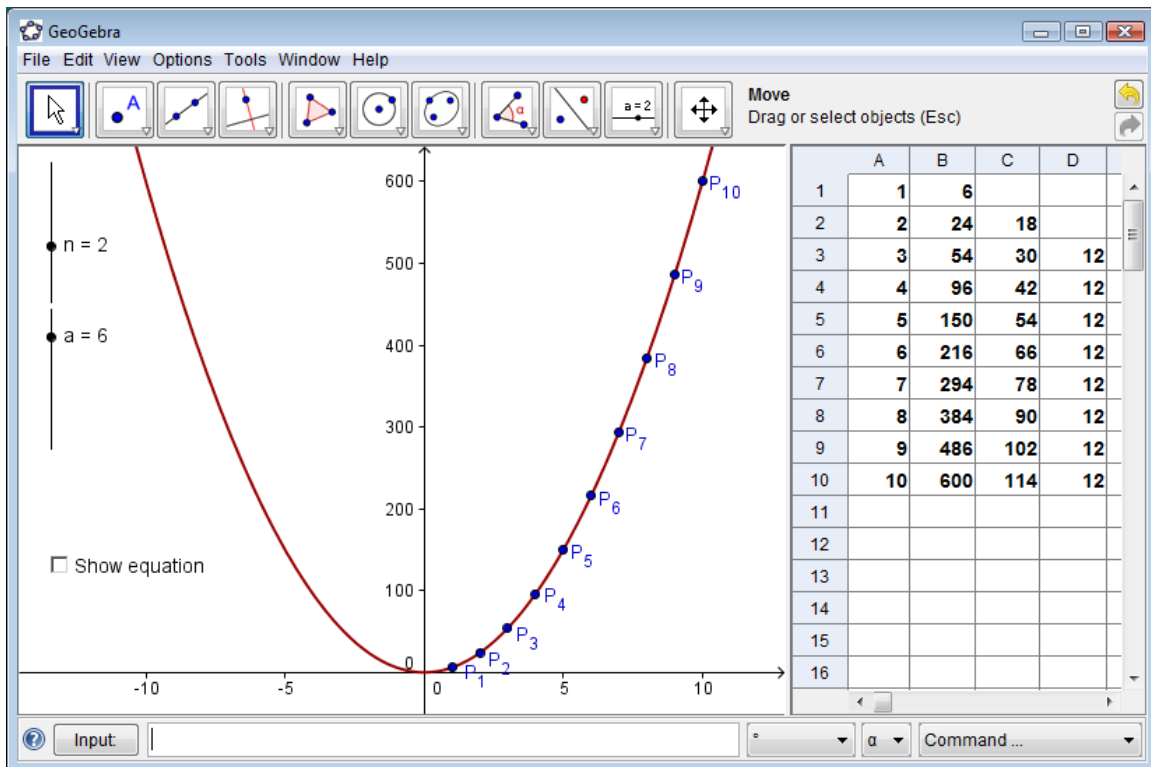
Instructions

Create a Scatter Plot from your Data

1	Enter the following numbers into the spreadsheet cells of column *A*: *A1*: 1 *A2*: 2
2	Highlight cells *A1* and *A2*. Relative copy the values to cell *A10* in order to create a sequence of different edge lengths. Hint: This creates the integers from 1 to 10.
3	In column *B*, enter the surface values you calculated earlier next to the corresponding edge length of the cube. Hint: You may collaborate with your neighbors to complete the table.
4	Create a Scatter Plot from this data: (1) Use the mouse to highlight all cells of columns *A* and *B* that contain numbers. (2) Right click (MacOS: *Ctrl*-click) on one of the highlighted cells and select 'Create List of Points' from the appearing *Context Menu*. Note: The values in column *A* determine the *x*-coordinates and the values in column *B* specify the *y*-coordinates of the plotted points. Hint: The points created from the data are displayed in the *Algebra View* as a list of points. By default, GeoGebra calls this list L_1.
5 ⬌	Use tool *Move Drawing Pad* in order to change the scale of the *y*-axis so that all points are visible in the *Graphics View*. Hint: Select tool *Move Drawing Pad.* Click on the *y*-axis and drag it down until you can see the 600 tick mark.

Investigate the Number Pattern in Column B

|	
6	In cell *C2*, enter the formula = B2 - B1 to compute the difference of the two successive surface values. Hint: After entering the equal sign, you may click on cell *B2* in order to enter its name into the active cell *C2*.
7	Select cell *C2* and relative copy the formula down to cell *C10*.
8	In cell *D3*, enter the formula = C3 - C2 to compute the difference of the two successive differences.

9	Select cell *D3* and relative copy the formula down to cell *D10*.

Task 1

Examine the number sequences in columns *C* and *D*. Make a conjecture about the polynomial function that runs through all points plotted in the *Graphics View* and allows you to compute the surface of a cube for any given edge length *e*.

- Is it possible to determine the degree of this polynomial by investigating the sequences of differences you generated in columns *C* and *D*?
- Explain to your neighbor why we were repeatedly calculating differences of successive values and what they actually mean.
- Is it possible to determine the coefficient of the polynomial by investigating the sequences of differences you generated in columns *C* and *D*?
- Would this also work if the values in column *A* are not successive integers (e.g., 1, 3, 5,…)? Explain your answer.

Check your Conjecture about the Polynomial

10	a = 2	Create a slider *n* with interval from 0 to 5 and increment 1. Change the orientation of the slider from 'Horizontal' to 'Vertical'.
11	a = 2	Create a slider *a* with interval from -10 to 10 and increment 1. Change the orientation of the slider from 'Horizontal' to 'Vertical'.
12		Enter the polynomial `f(x) = a * x^n` in order to create a polynomial of degree *n* with coefficient *a*. Note: Both, the degree *n* as well as the coefficient *a* can be changed by using the corresponding sliders.
13		Change the values of sliders *a* and *n* to match your conjecture. Does the polynomial run through all points plotted in the *Graphics View*?

Enhance your Construction

| 14 | ABC | Insert the polynomial's equation as a dynamic text in the *Graphics View*.

Hint: Select tool *Insert Text* and click on the *Graphics View* to open the text edit dialog window.
(1)   Enter `f(x)=` into the text edit dialog window.
(2)   Click on the graph of the polynomial to insert it's name into the text edit dialog window.
       Note: GeoGebra will enter the syntax necessary for dynamic text automatically.
(3)   Click on the *OK* button. |

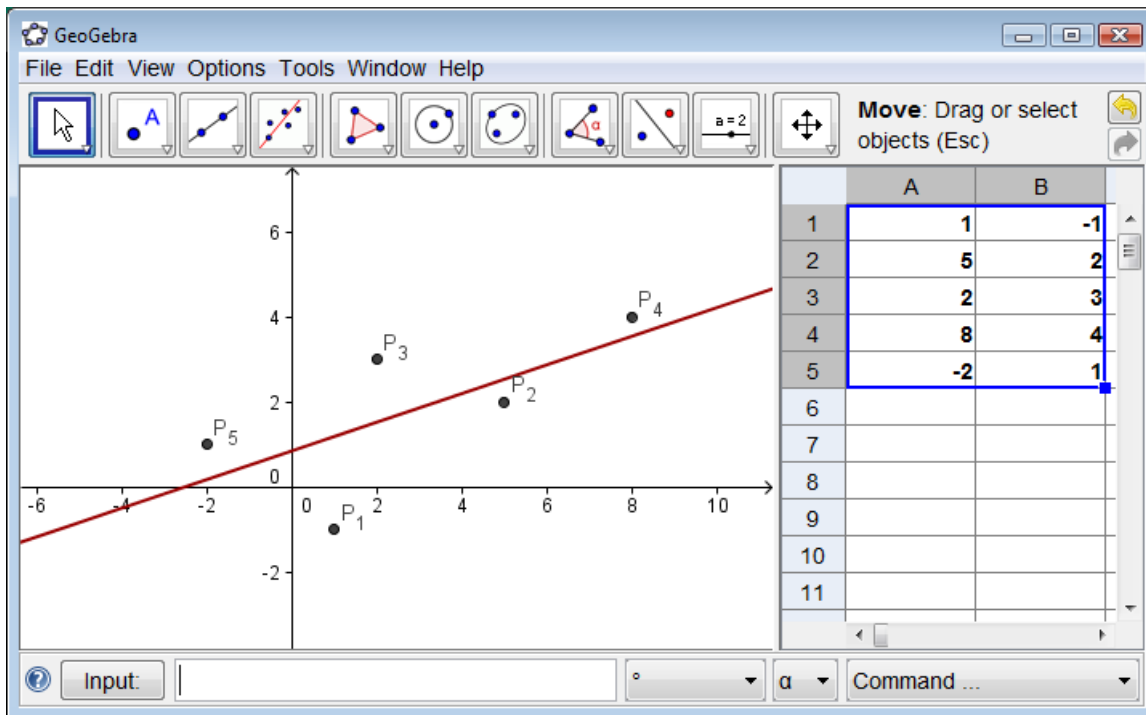| 15 | ☑ | Insert a checkbox that allows you to show/hide the polynomial's equation. |
		Hint: Select tool *Checkbox to Show/Hide Objects* and click on the *Drawing Pad* to open the checkbox dialog window.
		(1) Enter the caption `Show equation`.
		(2) Click on the little arrow to open list of available objects.
		(3) Select *text1* from this list and click the 'Apply' button.
16		Activate the *Move* tool and try out if your checkbox controls the visibility of the text.
17		Open the Properties Dialog and enhance the layout of the objects in the *Graphics View* (e.g., change the color of the polynomial and the points, match the color of the text with the color of the polynomial, fix the position of the slider, checkbox, and text in the *Graphics View*).

Task 2

- Try if this concept of investigating sequences of differences of two successive function values works for all polynomials $f(x) = a\, x^n$.
  Hint: You may enter a formula into cell *B1* and relative copy it down to cell *B10* in order to create a list of function values. Don't forget to start the formula with an equal sign (e.g., `= x^2`)

- What modifications in the *Spreadsheet View* and *Graphics View* are necessary to be able to easily determine the constant of polynomials $f(x) = a\, x^n + b$?

5. Scatter Plot and Best Fit Line

Preparations

- Open a new GeoGebra file.
- Show the Spreadsheet View (*View* menu).
- Hide the *Algebra View* (*View* menu).
- In the *Options* menu set the *Labeling* to 'New Points Only'*.*

Instructions

1	Enter the following numbers into the spreadsheet cells of column *A*: *A1*: 1 *A2*: 5 *A3*: 2 *A4*: 8 *A5*: -2
2	Enter the following numbers into the spreadsheet cells of column *B*: *B1*: -1 *B2*: 2 *B3*: 3 *B4*: 4 *B5*: 1
3	Create a Scatter Plot from this data: (1) Use the mouse to highlight all cells of columns *A* and *B* that contain numbers. (2) Right click (MacOS: *Ctrl*-click) on one of the highlighted cells and select 'Create List of Points' from the appearing *Context Menu*. <u>Note</u>: The values in column *A* determine the *x*-coordinates and the values in column *B* specify the *y*-coordinates of the plotted points.
4	Use tool *Best Fit Line* in order to create the line that best fits your data points. <u>Hint</u>: Activate tool *Best Fit Line* and select all data points using a selection rectangle: Click in the upper left corner of the *Graphics View*. Hold the mouse key down while moving the pointer to the lower right corner of the *Graphics View* in order to specify the

	selection rectangle.
5	Change color and thickness of the line using the *Properties Dialog*.
6	Using this construction you can easily demonstrate how outliers impact the best fit line of a data set:
	Drag one of the points with the mouse and explore how this modification influences the best fit line.
	Note: You can easily change the initial data in the *Spreadsheet View* as well.

Importing Data from other Spreadsheets

Note: GeoGebra allows you to copy and paste data from other spreadsheet software into the GeoGebra spreadsheet:
- Select and copy the data you want to import (e.g., use the keyboard shortcut *Ctrl-C* in order to copy the data to your computer's clipboard). Note: You need to press both keys (e.g., *Ctrl*-key and *C*-key) at the same time in order to apply a keyboard shortcut.
- Open a GeoGebra window and show the *Spreadsheet View*.
- Click on the spreadsheet cell that should contain the first data value.
- Paste the data from your computer's clipboard into GeoGebra's *Spreadsheet View*. In order to do so, you can either use the keyboard shortcut *Ctrl-V* or right click (MacOS: *Ctrl*-click) on the highlighted cell and select 📋 *Paste* from the appearing *Context Menu*.

6. Exploring Basic Statistics Commands

Yesterday, you gave a mathematics quiz to the 25 students of your 1st period math class. After the quiz, you asked your students to rate the difficulty of the quiz on a scale from 1 ('very easy') to 5 ('very difficult').
- 4 of your students rated the quiz 'very easy' (1)
- 6 students rated the quiz 'easy' (2)
- 6 other students rated the quiz 'difficult' (4)
- 1 student rated the quiz 'very difficult' (5)
- The rest of the students thought the difficulty of the quiz was 'ok' (3).

Preparations

- Open a new GeoGebra file.
- Show the Spreadsheet View (*View* menu).
- Show the *Algebra View* (*View* menu).

- Drag the origin of the coordinate system to the lower left corner of the *Graphics View* using tool ✛ *Move Drawing Pad*.

Task 1: Create a Histogramm

Enter the data into GeoGebra's spreadsheet and create a histogram.
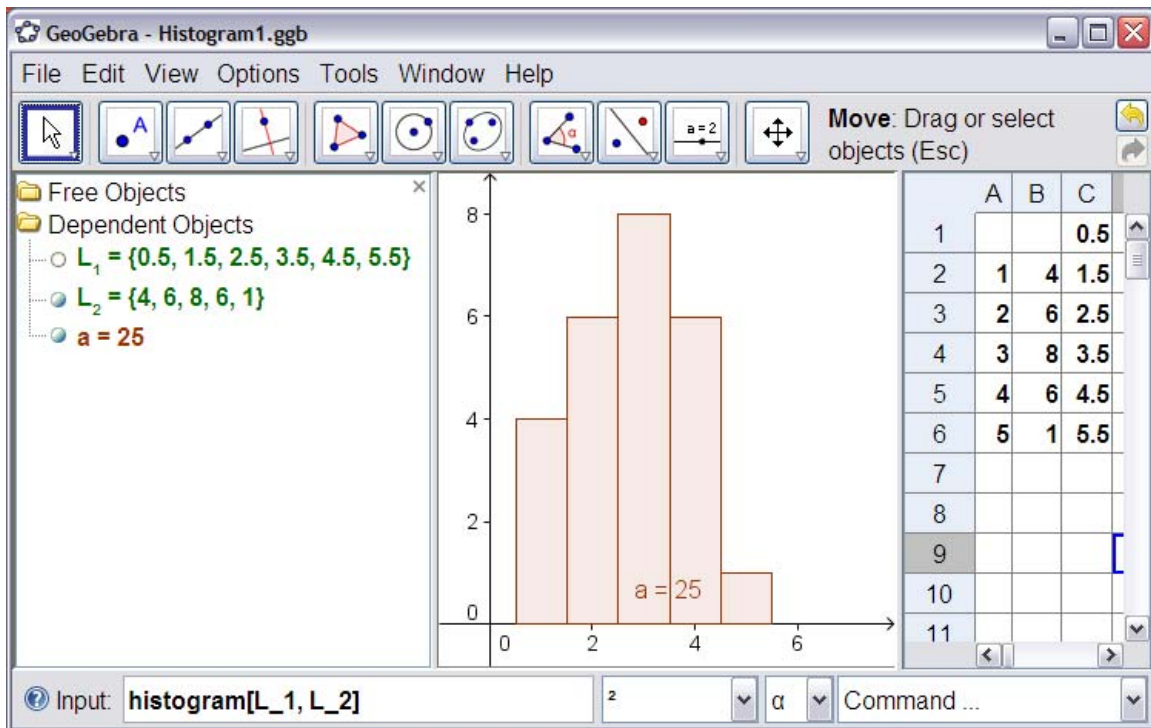
Instructions

1	Enter the rating scores from 1 to 5 into column *A* of the spreadsheet, starting in cell *A2*.
2	Enter the number of students who rated the quiz with each score into column *B* of the spreadsheet, starting with cell *B2*.

3	Enter the class boundaries (starting and ending values of the histogram bar widths) into column *C*, starting in cell *C1*. Make sure that the histogram bars are drawn symmetrically around the rating scores (i.e., numbers from 1 to 5 on the *x*-axis).
	
	Hints:
	   • The first bar is supposed to be symmetrically drawn around the *x*-value 1. Therefore, the bar should start at 0.5 and end at 1.5.
	   • Enter the value 0.5 in cell *C1* and the value 1.5 into cell *C2*.
	   • Relative copy these two values down to cell *C6*. This creates the

	missing values 2.5, 3.5, 4.5, and 5.5.
4	Create a list of numbers from the class boundary values in column *C*. (1) Use the mouse to highlight all cells of columns *A* and *B* that contain numbers. (2) Right click (MacOS: *Ctrl*-click) on one of the highlighted cells and select 'Create List' from the appearing *Context Menu*. <u>Note</u>: In the *Algebra View*, the list $L_1 = \{0.5, 1.5, 2.5, 3.5, 4.5, 5.5\}$ is displayed.
5	Create a list of numbers (L_2) from the values in column *B*.
6	Create a histogram using the command `Histogram[L_1, L_2]` with L_1 and L_2 being the two lists of numbers you created in steps 4 and 5.

Task 2: Determine mean, median, and mode

1. Make a prediction for mean, median, and mode of the data you collected. <u>Hint</u>: You may use command `Sort[L_2]` in order to sort the list of frequencies of students who rated the difficulty of the quiz in each category.
2. Check your prediction using the commands *Mean*, *Median*, and *Mode*. <u>Hint</u>: If you don't know how to use these commands, just enter the command into the *Input Bar* and press the *F1* key in order to access the *Input Bar Help*.