Common Grounds for Modeling Mathematics in Educational Software

Introduction to the Special Track "Convergence on Math Assistants"

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Outline

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MAs and Doing Mathematics

Do mathematics at high school . . .

Computer Support	Categories of Doing
	(1) Model:
Simulation tools	identify objects, relations,
Coach, InLot	determine methods, . . .
	(2) Operate:
CAS	calculate, simplify, solve,
	differentiate, integrate, . . .
CAS: function graphs	**(3) Interpret:**
DGS !	place results in context
Spreadsheets	relate (recur ?) to (1)
	(4) Communicate:
Presentation tools	present, discuss,
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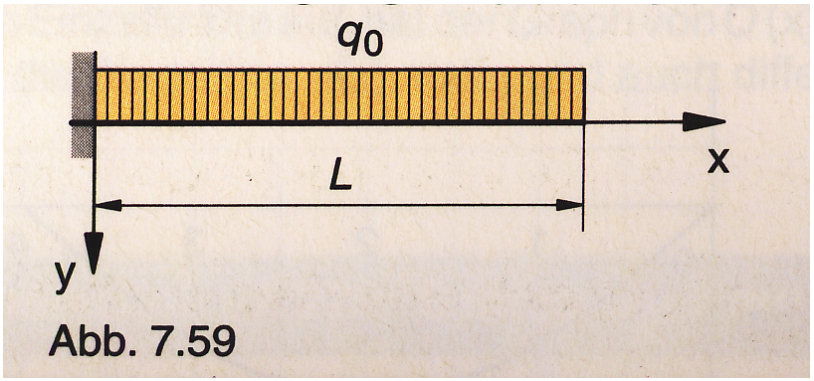
Example: Bending Lines

From a textbook for Technical High Schools (HTL)

Determine the beding line of a beam of length L, which consists of homogenous material, which is clamped on one side and which is under constant line load q_0.

Hint: Use the constraints $y(0) = 0$, $y'(0) = 0$, $V(0) = q_0 \cdot L$, $M_b(L) = 0$.



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. . . intuitively, informally		. . . formally
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Simulation tools Coach, InLot	identify objects, relations, determine methods, . . .	specify formally
	(2) Operate:	prove theorems ?
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A "Step" in doing math

A *step* starts from a *Context* and produces a result which can be **justified** . . .

$$step : Context \times State \times Interact \longrightarrow Context \times State \times Result$$

. . . where *State* concerns technicalities of MAs and

*Interact*ion: compound operation

- draw a geometric object (e.g. ortho-center of a triangle)
- call a CAS command (e.g. *Integrate* $x^3 + x^2 + x + 1$ *dx*)
- . . .

atomic operation

- substitute a value for a variable
- apply a rule (e.g. $\int 2x\ dx = x^2 + c$) to transform a formula
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Formality of *Context* constrains rigor of justification !

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Formal specification

Specification of the problem on the bending line:

$$\begin{aligned} \textit{in} &: \textit{length } L, \textit{function } q_0 \\ \textit{pre} &: L > 0 \ \wedge \ q_0 \textit{ is\_integrable\_in } x \\ \textit{out} &: \textit{function } y(x) \\ \textit{post} &: y(0) = 0 \wedge y'(0) = 0 \ \wedge \ V(0) = q_0.L \ \wedge \ M_b(L) = 0 \end{aligned}$$

where V and M_b are constant function symbols in the theory of "bending lines".

Formal Specification required for *mechanical* steps !

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The human part in formulas

(i) Problem solving creates a *Result*:

$$solve : Theory \times Context \times Specification \longrightarrow Context \times Result$$

where
$Specification = Input \times Precondition \times OutputVar \times Postcondition$
and $post(in, res)$ holds for $pre(in)$

(ii) Theorem proving constructs a *Theorem*:

$$prove : Theory \times Context \times Predicate \longrightarrow Theory \times Theorem$$

(i) expands knowledge *outside* the formal model - "applied mat"
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1. Convergence on concepts for learning with MAs ?
   - *Step is a basic notion, less or more formal !*
   - . . .
2. Convergence on technology of MAs ?
   - *Serve MAs with Logic-based math-engines !*
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3. Convergence on principles of e-learning ?
   - *We need a formal domain model of e-learning !*
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