

Teaching logic using ProofWeb

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Outline

Motivation

- Teaching logic with a computer
- Web interface for proof assistants

ProofWeb

- Coq
- Display Styles
- Working with ProofWeb

MathWiki

- Project
- Comparison with QED
- Conclusion

Motivation: teaching logic with the computer

logic course for math/computer science students :

propositional logic

predicate logic

predicate logic with equality

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practising natural deduction proofs

- **on paper**

- **with the computer**

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both necessary: complement each other

Web interface for proof assistants

- ▶ **No installation** for a user (not even browser plug-in)
- ▶ **Responsive**, fast interaction
- ▶ Resembles and behaves like a local interface
- ▶ Multiple proof assistants (like **ProofGeneral**)
 - ▶ Updated on the server, with extensions
- ▶ **Secure** environment

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ProofWeb

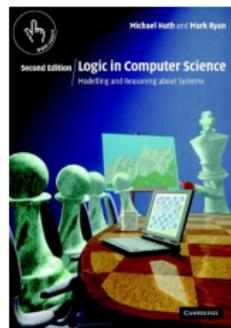
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ProofWeb features

- built on top of serious proof system: Coq
 - ▶ students work with an industrial strength system
 - ▶ proofs look exactly like in a traditional textbookcompatible with: Huth & Ryan, *Logic in Computer Science*
- web-based
 - ▶ students don't need to install anything
 - ▶ students can access their work from anywhere
 - ▶ teacher has at all times full info on student's work
- comes with a manual explaining the system
- comes with a set of graded exercises



Coq

proof assistant based on constructive logic
developed at INRIA, France
1984 until today

used for impressive proofs :

- **four color theorem**, Georges Gonthier
- **verified C compiler**, Xavier Leroy

power of Coq also makes **ProofWeb** attractive for education

natural deduction (Fitch style)

1	$\exists x(P(x) \vee \neg Q(a))$	assumption
2	$Q(a)$	assumption
3	$b \quad P(b) \vee \neg Q(a)$	assumption
4	$P(b)$	assumption
5	$\exists x P(x)$	$\exists i$ 4
6	$\neg Q(a)$	assumption
7	\perp	$\neg e$ 6,2
8	$\exists x P(x)$	$\perp e$ 7
9	$\exists x P(x)$	$\vee e$ 3,4—5,6—8
10	$\exists x P(x)$	$\exists e$ 1,3—9
11	$Q(a) \rightarrow \exists x P(x)$	$\rightarrow i$ 2—10

natural deduction (Fitch style)

1	H1: $\exists x, (P x \vee \neg Q a)$	assumption
2	H2: $Q a$	assumption
	b	
3	H3: $P b \vee \neg Q a$	assumption
4	H4: $P b$	assumption
5	$\exists x, P x$	$\exists i$ 4
6	H5: $\neg Q a$	assumption
7	\perp	$\neg e$ 6,2
8	$\exists x, P x$	$\perp e$ 7
9	$\exists x, P x$	$\vee e$ 3,4-5,6-8
10	$\exists x, P x$	$\exists e$ 1,3-9
11	$Q a \rightarrow \exists x, P x$	$\rightarrow i$ 2-10
12	$\exists x, (P x \vee \neg Q a) \rightarrow Q a \rightarrow \exists x, P x$	$\rightarrow i$ 1-11

1	$\exists x(P(x) \vee \neg Q(a))$	assumption
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3	b $P(b) \vee \neg Q(a)$	assumption
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user input

```
Require Import ProofWeb.
```

```
Variable P Q : D -> Prop.
```

```
Variable a : D.
```

```
Theorem example :  $\exists x, (P(x) \wedge Q(a)) \rightarrow Q(a) \rightarrow \exists x, P(x)$ .
```

```
Proof.
```

Qed.

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Proof.
```

```
  imp_i H1.
```

```
  imp_i H2.
```

```
  f_exi_e H1 b H3.
```

```
  f_dis_e H3 H4 H5.
```

```
  f_exi_i H4.
```

```
  fls_e.
```

```
  f_neg_e H5 H2.
```

```
Qed.
```

Working with ProofWeb

The screenshot shows a web browser window with the address bar containing `http://proofweb.cs.ru.nl/index.html`. The browser's menu bar includes File, Edit, View, Web, Go, Bookmarks, Tabs, and Help. The toolbar contains Back, Forward, Home, Refresh, and Stop buttons. Below the toolbar is a navigation menu with icons for navigation and a list of menu items: File, Display, Templates, Backward, Forward, Query, Debug, Help, and Logout.

The main content area is split into two columns. The left column contains the following text:

```
(* Exercise 1 *)  
Require Import ProofWeb.  
Variables A B : Prop.  
Theorem prop_001 : (A ∧ B) -> A.  
Proof.  
  inp_i H.  
Qed.
```

The right column displays the output of the theorem prover, showing a subgoal and a proof step:

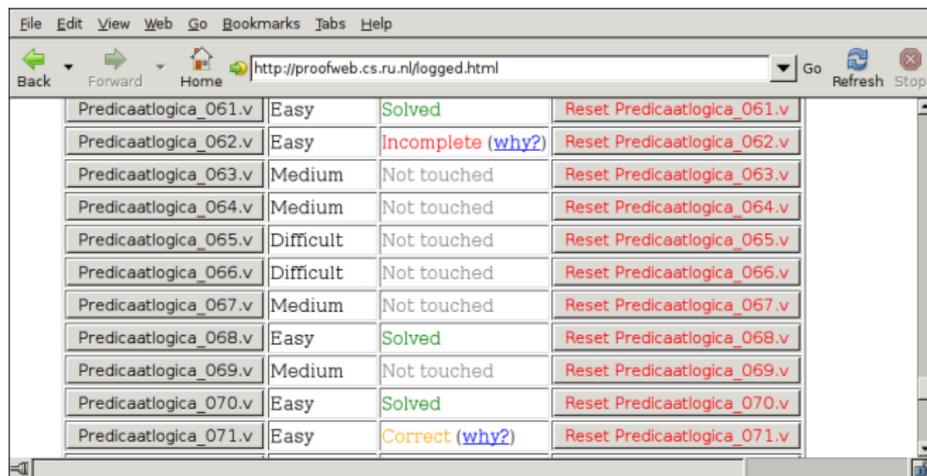
```
1 subgoal  
  H : A ∧ B  
  =====  
  A  
  
  ...  
  A  
----- +i[H]  
A ∧ B → A
```

The browser's status bar at the bottom shows the current page URL: `http://proofweb.cs.ru.nl/i108/course_i108.html`.

exercise colors

possibilities for an exercise :

- Not touched
- **Incomplete**
- **Correct**
- **Solved**



The screenshot shows a web browser window with the address bar displaying "http://proofweb.cs.ru.nl/logged.html". The browser's menu bar includes "File", "Edit", "View", "Web", "Go", "Bookmarks", "Tabs", and "Help". The browser's navigation buttons (Back, Forward, Home) and the address bar are visible. The main content area displays a table with 11 rows, each representing an exercise. The table has four columns: the exercise ID, the difficulty level, the current status, and a "Reset" button. The status column uses color-coding: green for "Solved", red for "Incomplete", orange for "Correct", and black for "Not touched".

Predicaatlogica_061.v	Easy	Solved	Reset Predicaatlogica_061.v
Predicaatlogica_062.v	Easy	Incomplete (why?)	Reset Predicaatlogica_062.v
Predicaatlogica_063.v	Medium	Not touched	Reset Predicaatlogica_063.v
Predicaatlogica_064.v	Medium	Not touched	Reset Predicaatlogica_064.v
Predicaatlogica_065.v	Difficult	Not touched	Reset Predicaatlogica_065.v
Predicaatlogica_066.v	Difficult	Not touched	Reset Predicaatlogica_066.v
Predicaatlogica_067.v	Medium	Not touched	Reset Predicaatlogica_067.v
Predicaatlogica_068.v	Easy	Solved	Reset Predicaatlogica_068.v
Predicaatlogica_069.v	Medium	Not touched	Reset Predicaatlogica_069.v
Predicaatlogica_070.v	Easy	Solved	Reset Predicaatlogica_070.v
Predicaatlogica_071.v	Easy	Correct (why?)	Reset Predicaatlogica_071.v

trying ProofWeb

three possibilities :

<http://proofweb.cs.ru.nl/>

1. **guest access** (no registration needed)
2. **host course in Nijmegen** (free)
3. download (open source) and **host course on your own system**

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A future project: MathWiki

- ▶ 4 year project, 2009-2013
 - ▶ Post-doc and PhD student
- ▶ Combine an **encyclopedia** with a **proof assistant** environment
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 - ▶ Semantically annotated high-level knowledge
- ▶ Web-based, interactive, **collaborative** environment
 - ▶ For multiple proof assistants
- ▶ Supports large joint formalisations in a **distributed** way
- ▶ *Search and retrieval*
 - ▶ informal and formal
 - ▶ high level and proof assistant specific

Binomial coefficient - MathWiki - Iceweasel

File Edit View History Bookmarks Tools Help

http://mathwiki/Binomial_coefficient.html

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MathWiki

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Article Search

semantic search

Theorem Proof

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formalizations

http://mathwiki/mmm

Binomial coefficient

In **mathematics**, particularly in **combinatorics**, a **binomial coefficient** is a **coefficient** of any of the terms in the expansion of the **binomial** $(x+y)^n$. Colloquially given, say there are n pizza toppings to select from, if one wishes to bake a pizza with exactly k toppings, then the binomial coefficient expresses how many different types of such k -topping pizzas are possible.

Definition

Given a non-negative integer n and an integer k , the binomial coefficient is defined to be the natural number

$$\binom{n}{k} = \frac{n \cdot (n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 1} = \frac{n!}{k!(n-k)!} \quad \text{if } n \geq k \geq 0$$

and

$$\binom{n}{k} = 0 \quad \text{if } k < 0 \text{ or } k > n$$

where $n!$ denotes the **factorial** of n .

Definition in Coq (edit formalization)

```
Definition C (n p:nat) : R :=
  (fact n) / ((fact p) * (fact (n - p))).
```

Definition in Mizar (edit formalization)

```
definition
  let k,n be natural number;
  func n choose k means
  :: NEWTON:def 3
    for l be natural number st l = n-k holds
      it = (n!)/((k!) * (l!)) if n >= k
```

Tor Disabled

QED 15 years later?

- ▶ Success of the Wiki approach
 - ▶ Collaborative approach as a good way of developing bodies of shared knowledge
- ▶ *Semantic Web* technology can provide the presentation layer

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 - ▶ **Collaborative** approach as a good way of developing bodies of shared knowledge
- ▶ *Semantic Web* technology can provide the presentation layer
- ▶ Proof assistants provide mathematical semantics
 - ▶ **Solid systems**
 - ▶ Substantial formal developments
 - ▶ Coverage of a **wide range** of Proof Assistants:
 - ▶ initial proposed ones: Coq, Isabelle and Mizar
 - ▶ Type Theory, Higher Order Logic and Set Theory
 - ▶ classical and intuitionistic
 - ▶ de Bruijn style, LCF-style and batch-mode interaction

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- ▶ *Web 2.0*

Future plans

other proof display styles

other logics

- ▶ modal logics
- ▶ temporal logics
- ▶ logic in Dijkstra style

MathWiki